IBIS 2016

深層学習の基礎：自己組織化と教師付学習

---- 深層学習の理解に向けて：私の試み

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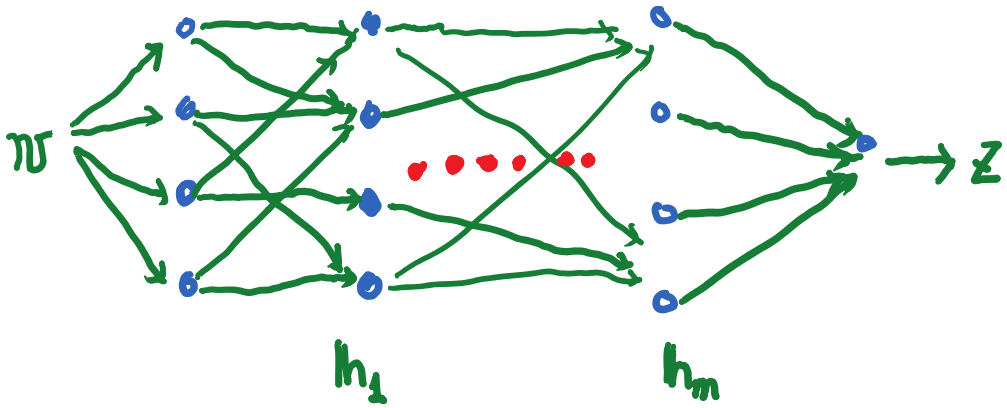
深層学習　自己組織化と確率降下教師付学習

RBM: 制約ボルツマン機械、オートエンコーダ、再帰結合回路



tricks !!

ideas !

ドロップアウト

雑音、コンボリューション

bi-directional

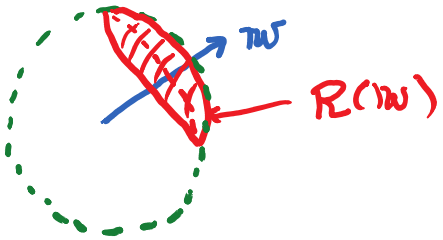
ニューロンのヘブ自己組織化

$h = f(w \cdot v - \theta)$: $p(\mathbf{v})$

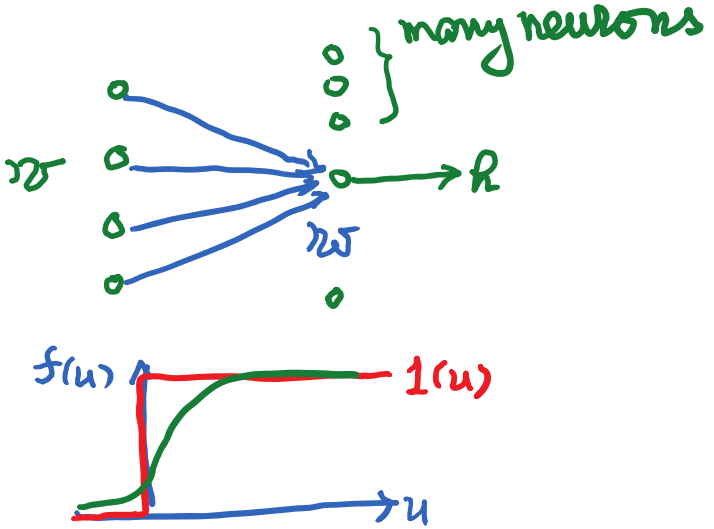
receptive field

$R(w) = \{ v \mid w \cdot v - v_0 > 0 \}$

$|v|^2 = \text{const}$



受容野



自己組織化学習

w : dynamics of $R(w)$

w : $p(v)$

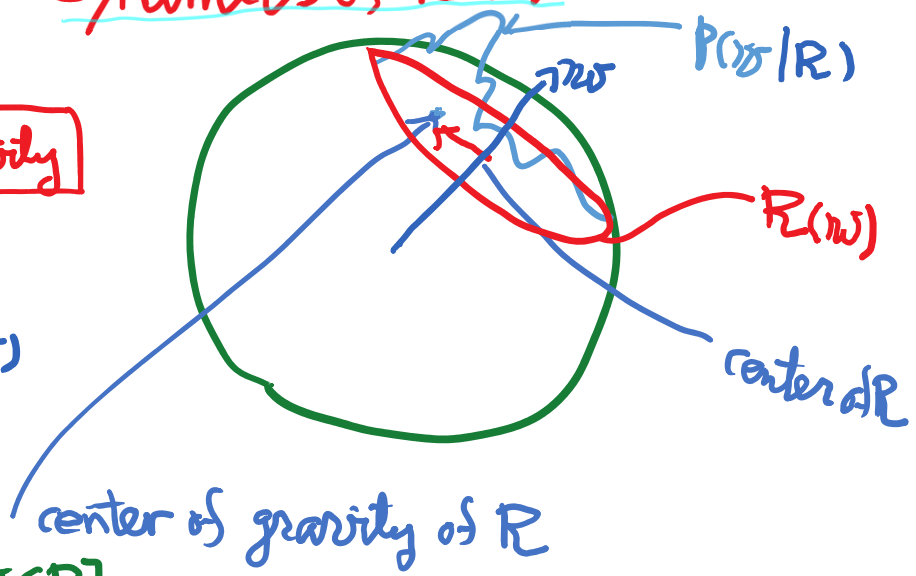
center ⇒ c. of gravity

$$\dot{w} = \langle h(-w + cv) \rangle_{p(v)}$$

$$\frac{1}{p_R}\dot{w} = -w + c\langle hv \rangle_R$$

$$\mathrm{Prob}\{v \in R\} = \langle 1(w \cdot v - v_0) \rangle_{p(v)}$$

$$E[hv \mid v \in R]$$



受容野の重心＝中心

学習の収束：平衡点

中心＝重心　大きさ τ

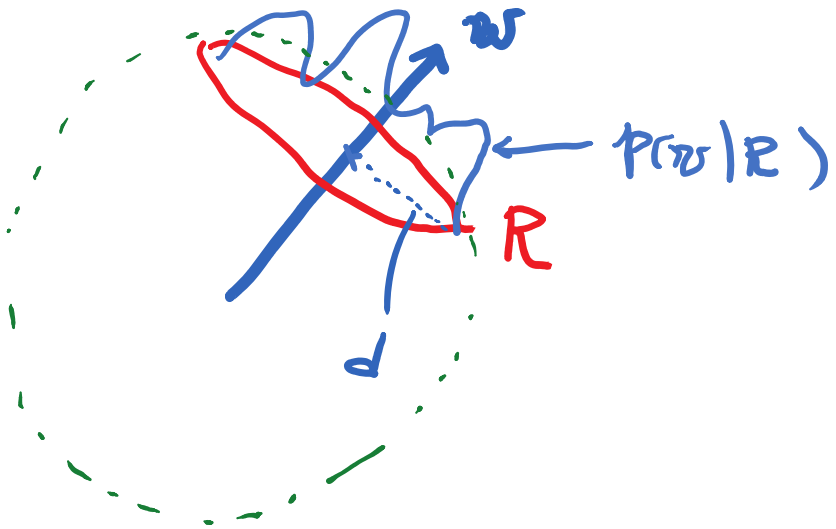
$$w = c\langle h w \rangle_R$$

↑ center ∝ c. of gravity

$w \cdot w = \tau$

d : radius of R(w)

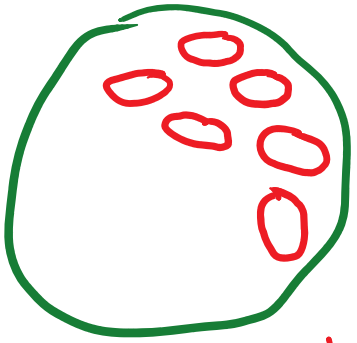
τ : small ⇔ d : large



例：単純な場合

1. $P(w) = c$: uniform

2. $P(w)$: single cluster

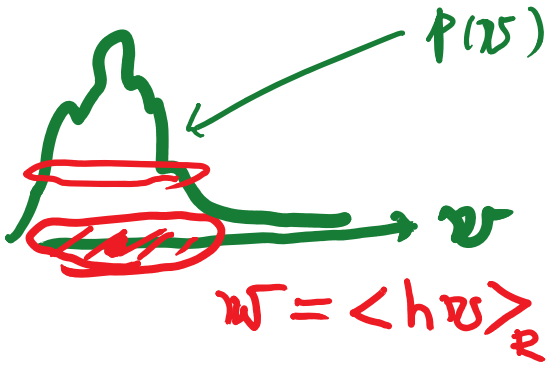


any w

$R(w)$

many neurons

h

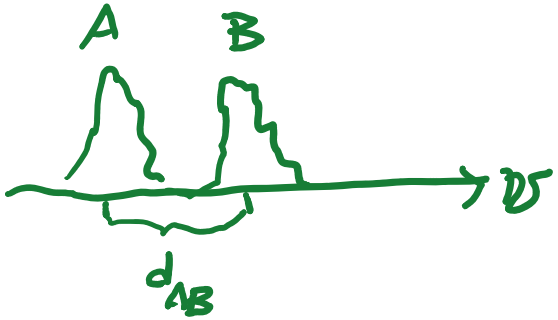


パターンのクラスター

1. two clusters

$d_{AB} > d_0$ $\quad w = c v_A,\ c v_B$

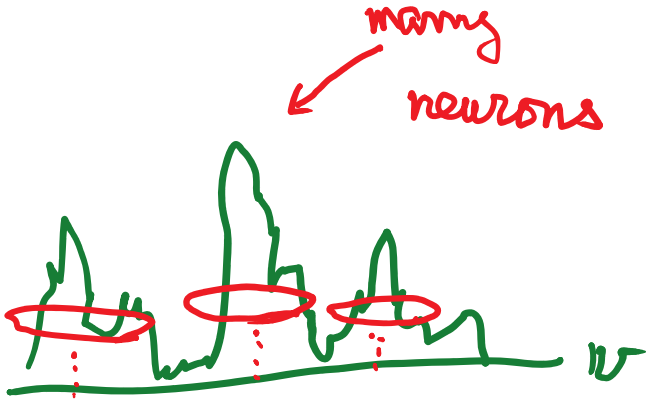
$d_{AB} < d_0$ $\quad w = c_1 v_A + c_2 v_B$



2. many clusters : multi-stable

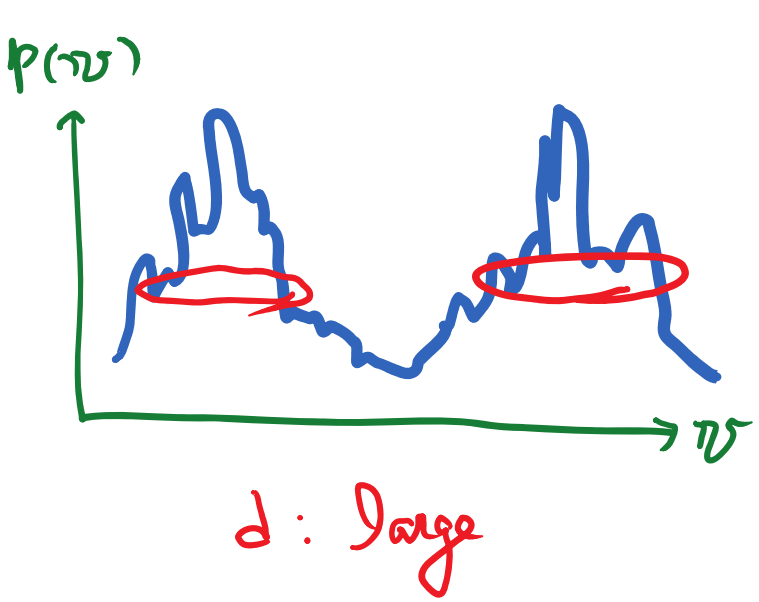
adequate size

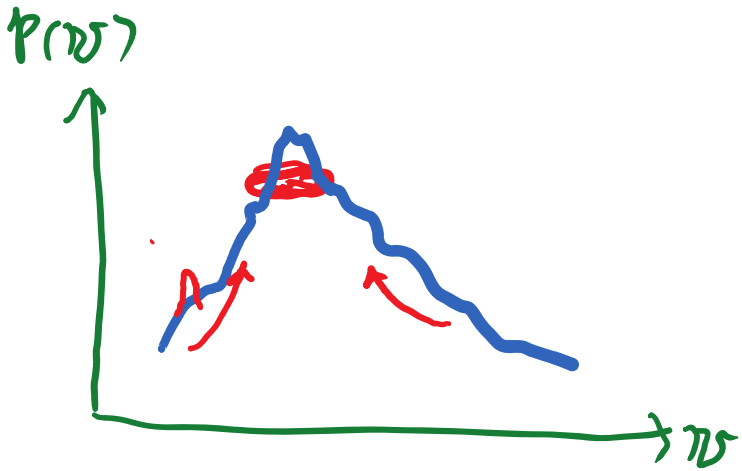
center = c. gravity



学習力学

$$\dot{w} = \langle k v \rangle_k - w \approx \nabla p(w) \quad : \ d \text{ small}$$





Lyapunov Function

$$F(w, v) = c\,(w \cdot v - \tau)^{+} - \frac{1}{2}|w|^2$$

$$\dot{w} = \eta \nabla_w F$$

$$u^{+} = \begin{cases} u, & u > 0, \\ 0, & u < 0 \end{cases}$$

$$\dot{F} = \nabla F \cdot \dot{w} = \eta |\nabla F|^2 > 0$$

$$\langle \dot{F} \rangle_{p(v)} = \int |\nabla F|^2 \, p(v|R) \, dv > 0$$

convergence to equilibria

Further Problems

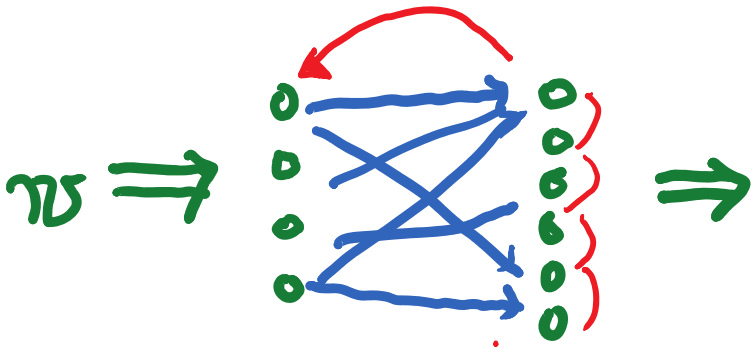
Dimension reduction; PCA, ICA

Distributed τ small clusters; large clusters

Mutual interactions among h-neurons neural field

feedback

Localized receptive fields invariance: convolution

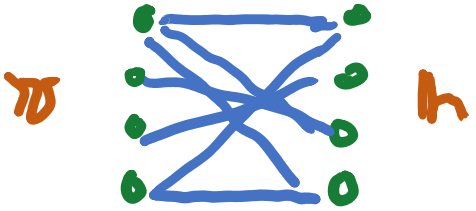


RBM: 制約ボルツマン機械

$$p(v,h) = \exp\{ b\cdot v + c\cdot h + h^T W v - \psi - \tfrac{1}{2}|v|^2 - \tfrac{1}{2}|h|^2 \}$$

$h^T W v = \sum h_i w_{ij} v_j$

$|v|^2 = \sum v_i^2$



$z = (v, h)$

energy machine :

b, c : neglect

自己組織化

$: p(\mathbf{v})$

$$\underset{W}{\text{minimize}} \; KL\left[p(v) : q_v(v; W) \right]$$

$$= \int p(v) \log \frac{p(v)}{q_v(v; W)} dv$$

$$\dot{w}_i = \varepsilon \{ -\langle h_i v \rangle_q + \langle h_i v \rangle_p \}$$

center w_i ?

center of gravity of $R(w_i)$

$$\langle hv \rangle_q = \frac{\partial}{\partial w} \psi(w)$$

ニューロン間の相互作用

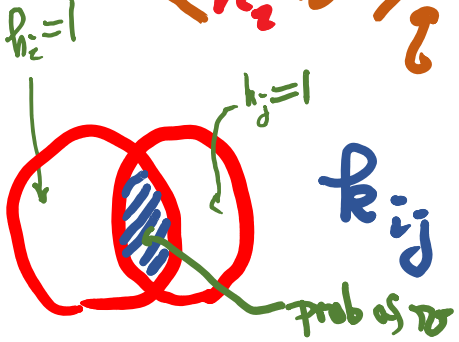
$$\langle h v \rangle_g = \frac{\partial \psi(W)}{\partial W} \quad ; \quad g(h,v) = \exp\{ h^T W v - \psi(W) \}$$

$-\frac{1}{2}|v|^2$

$$\psi(W) = \log \sum_h \int \exp\{ -\frac{1}{2}|v|^2 + h^T W v \} dv$$

$$= \log \sum_h \exp\{ \frac{1}{2}|h^T W|^2 \} + c$$

$$\langle h_i v \rangle_g = \sum R_{ij} w_j \quad : \text{interaction}$$



$$R_{ij} = E_g[h_i h_j]$$ ← joint firing prob

v: analog. Gaussian

受容野は周りのニューロンの受容野に影響される

$$w_j = \sum (\hat{k}^{-1})_{ji} \langle h_i v \rangle_g$$

center of gravity

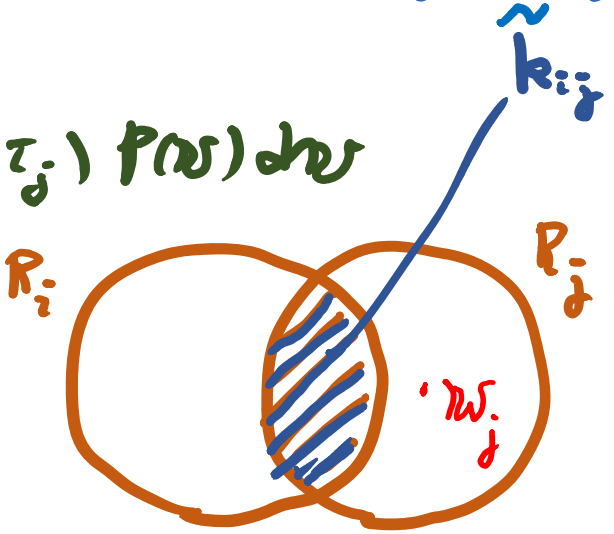
v : Gaussian

$$k_{ij} = \int f(w_i \cdot v - \tau_i) f(w_j \cdot v - \tau_j) P(v) dv$$

$$\langle h_i v \rangle_{g,\tau} = \sum \tilde{k}_{ij} w_j$$

center gravity

interaction

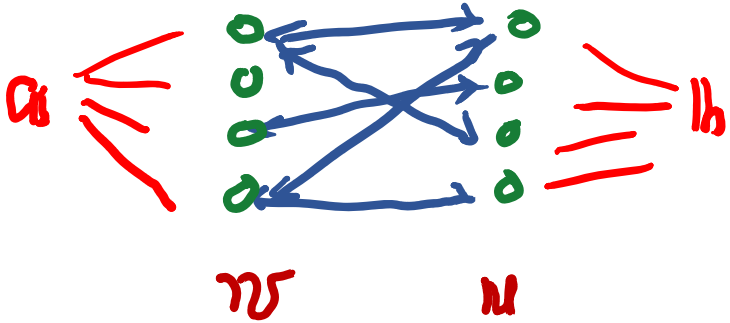


Recurrent Net (Auto-Encoder)

$$\begin{cases} \dot{u} = -u + f(W v) \\ \dot{v} = -v + f(S h) + a \end{cases}$$

multi-stable

$p(a)$



$$\dot{W} = -W + \varepsilon \langle h v^T \rangle_{p(a)}$$

$$\dot{S} = -S + \varepsilon' \langle v h^T \rangle_{p(a)}$$

$$S_{ij} = W_{ij} \quad \text{(equilibrium)}$$

ガウス型ボルツマン機械

$$g(v,h) = \exp\left\{-\frac{1}{2}|v|^2 - \frac{1}{2}|h|^2 + h^T W v - \psi(W)\right\}$$

$$\langle hv \rangle_p = WC \qquad C = \int v v^T p(v) dv$$

$\langle hv \rangle_p$ ← center of gravity

C ← covariance matrix

$$\langle hv \rangle_I = (I - WW^T)^{-1} W$$

$\langle hv \rangle_I$ ← interactions of k neurons

平衡状態の解

(唐木田 ; 東大)

$$WC = (I - WW^T)^{-1}W$$

General Solution $$W = U\,\mathrm{diag}\left(\sqrt{1-\frac{1}{\lambda_1}}, \ldots, \sqrt{1-\frac{1}{\lambda_m}}, 0, \ldots, 0\right) V$$

- othogonal matrix : U, V
- C diagonalized by V $\quad C = V^T\,\mathrm{diag}(\lambda_1, \ldots, \lambda_n)V$

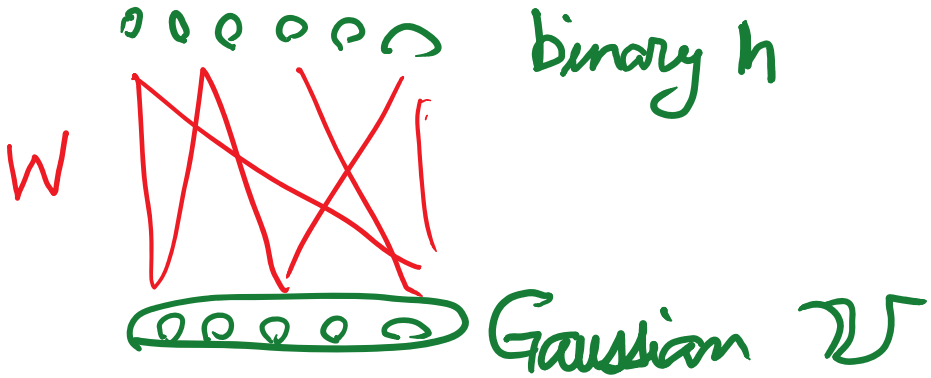
You can choose m(≤ k) eigen values form $\lambda_1, \ldots, \lambda_k > 1$

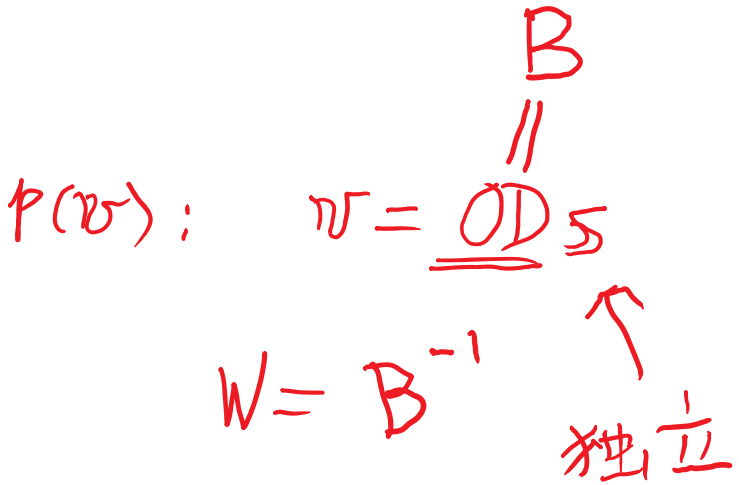
Stable Solution the case of m = k

ガウスーベルヌイ機械

ICA： 独立成分分析

唐木田





シミュレーションン例

The number of Neurons: N = M = 2, σ = 1/2

Sources p (s)

Uniform Distribution

Mixing

Input

$\mathbf{v} = B\mathbf{s}$

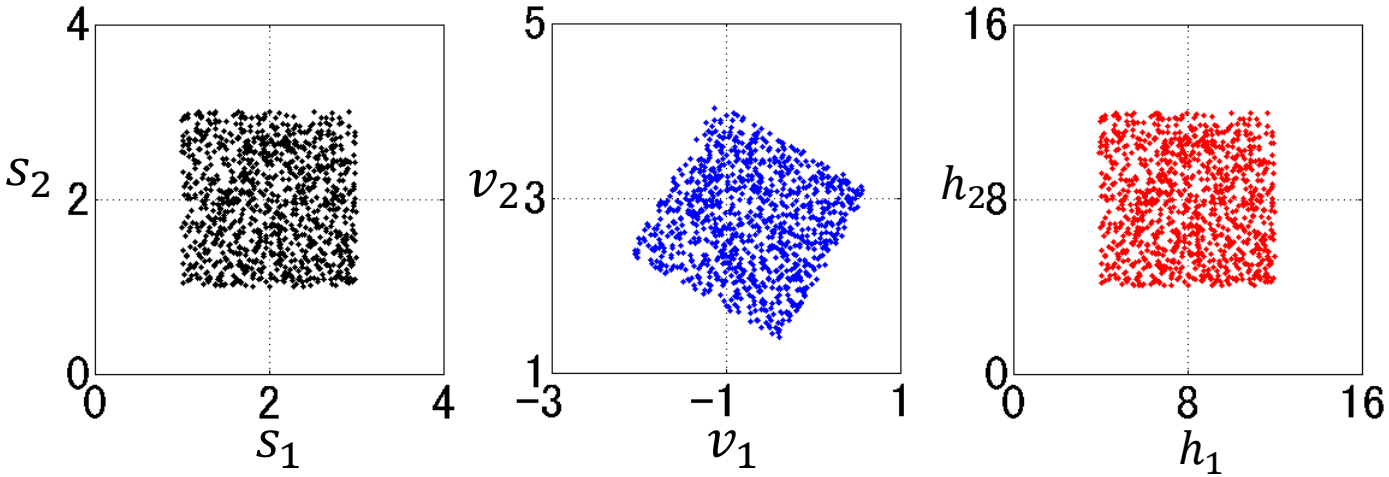
CD_1

ICA Solution

$\bar{W} = DB^T$

Output

$\mathbf{h} = \bar{W}\mathbf{v}$



Independent sources are extracted in G-B RBM

何が問題か：画像情報の構造、生成モデル（確率分布）

Uniform ： no structure

Aggregate of clusters ： Hebb self-organization

PCA ： Gaussian RBM　　submanifolds　　**：$p(\mathbf{v})$**

ICA ： Bernoulli-Gaussain　　sparse

スパース生成のクラスター（低次元）

階層構造と深層学習

invariancy

logical structure

hierarchies of hierarchy

大規模系の特徴

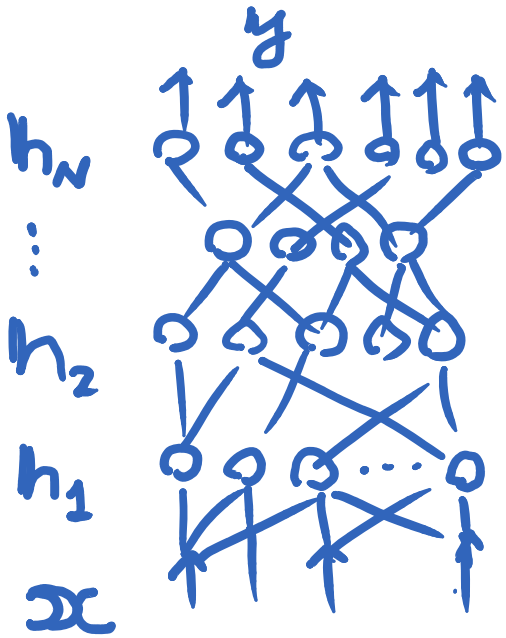
ランダム行列 A の固有値の分布

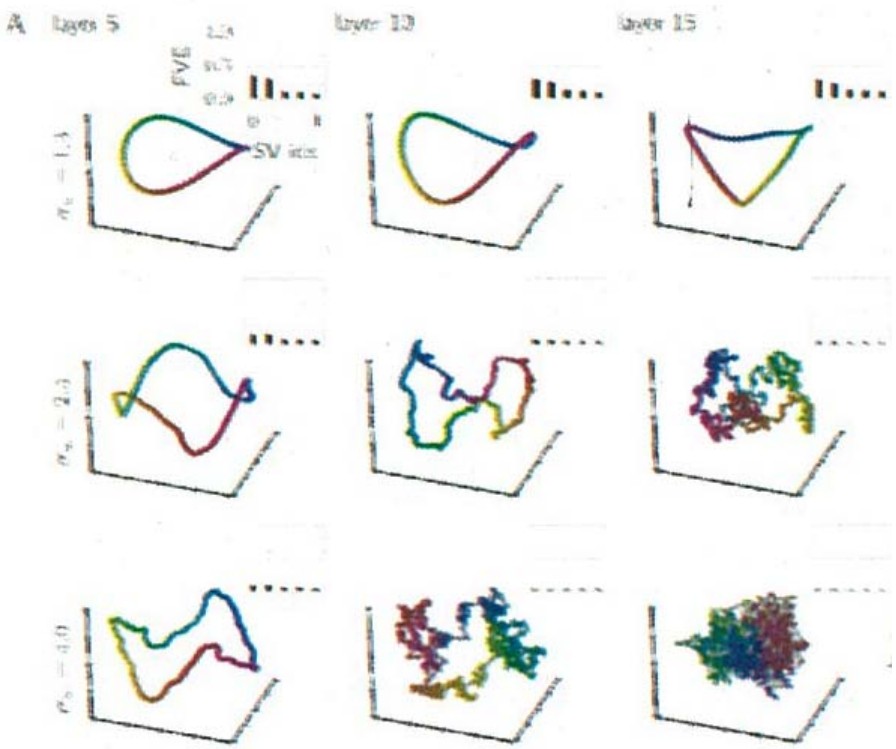
　　ほとんどが鞍点（極小解なし!!）

大規模回路

　　極小解は最小解の付近に集まる

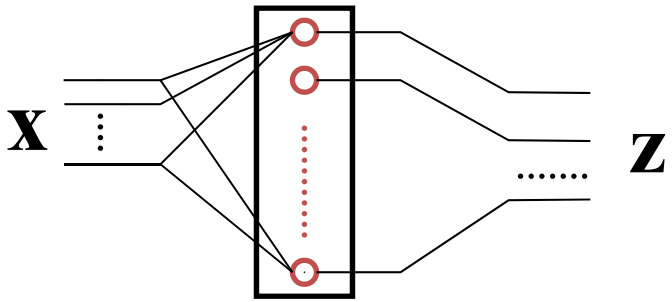
Poole et al (2016)
Deep neural networks





統計神経力学

1-layer network

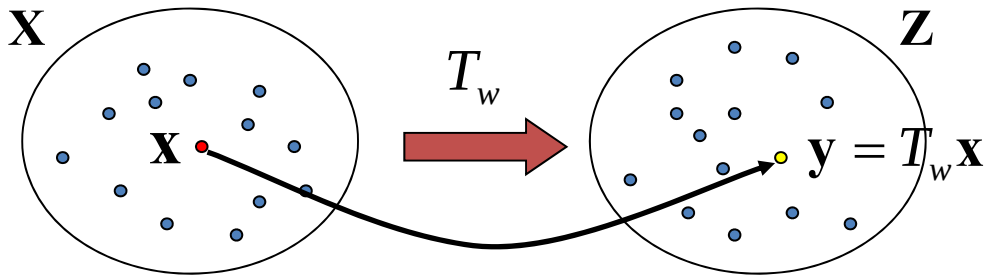


$$x_i = \pm 1$$

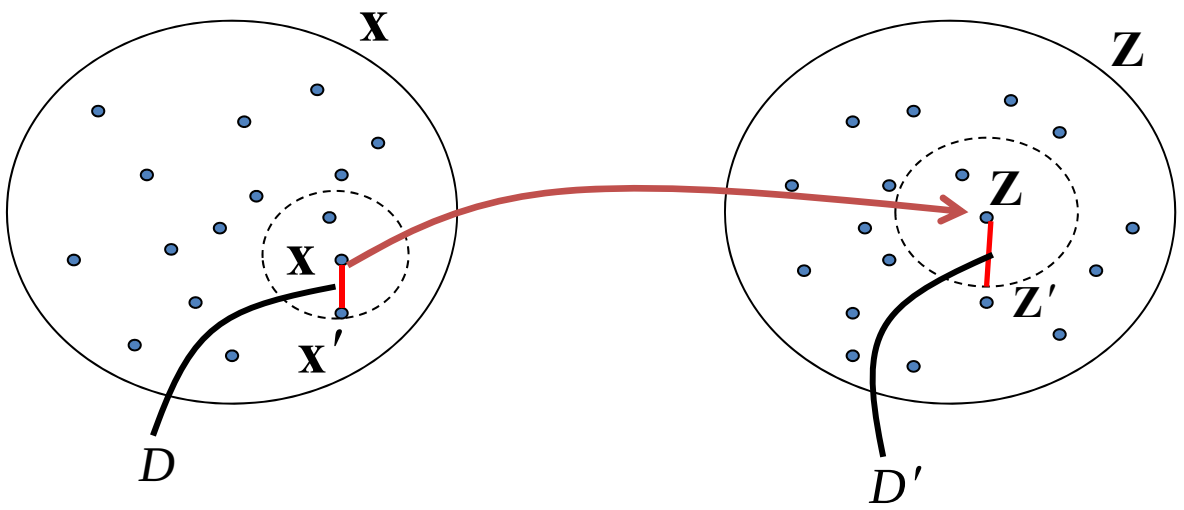
$$z_i = \pm 1$$

$$\mathrm{sgn}\left(\sum_{j=1}^{n} w_{ij} x_j\right)$$

$$\mathbf{z} = T_w \mathbf{x} = \mathrm{sgn}(W\mathbf{x})$$



距離法則：stability



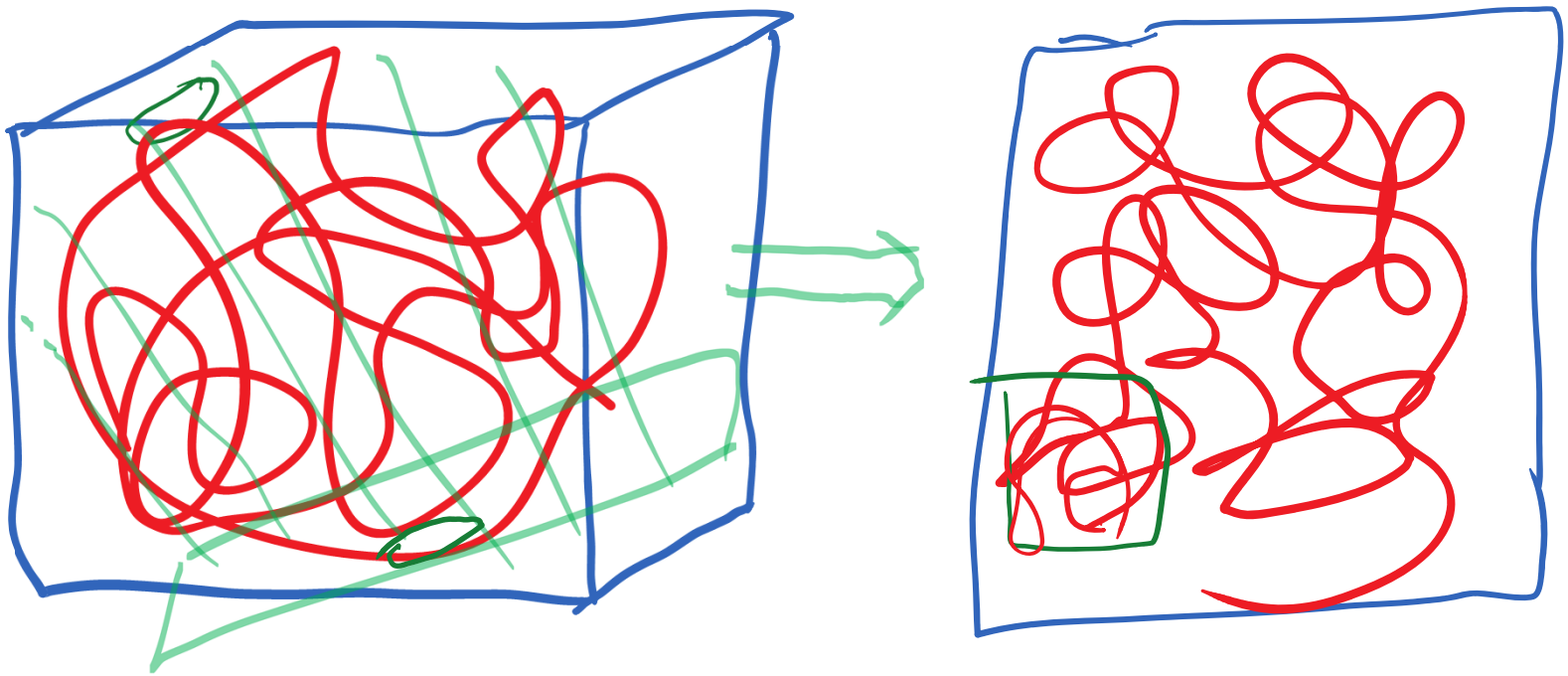
$$D(\mathbf{x},\mathbf{x}') = \frac{1}{2n}\sum\left|x_i - x'_i\right| = D$$

$$D\left(T_w\mathbf{x}, T_w\mathbf{x}'\right) = \phi(D) = D'$$

$$\begin{matrix}\| & \| \\ \mathbf{z} & \mathbf{z}'\end{matrix}$$

$$\phi(D) = \frac{2}{\pi}\sin^{-1}\left(c\sqrt{D}\right)$$

次元 小



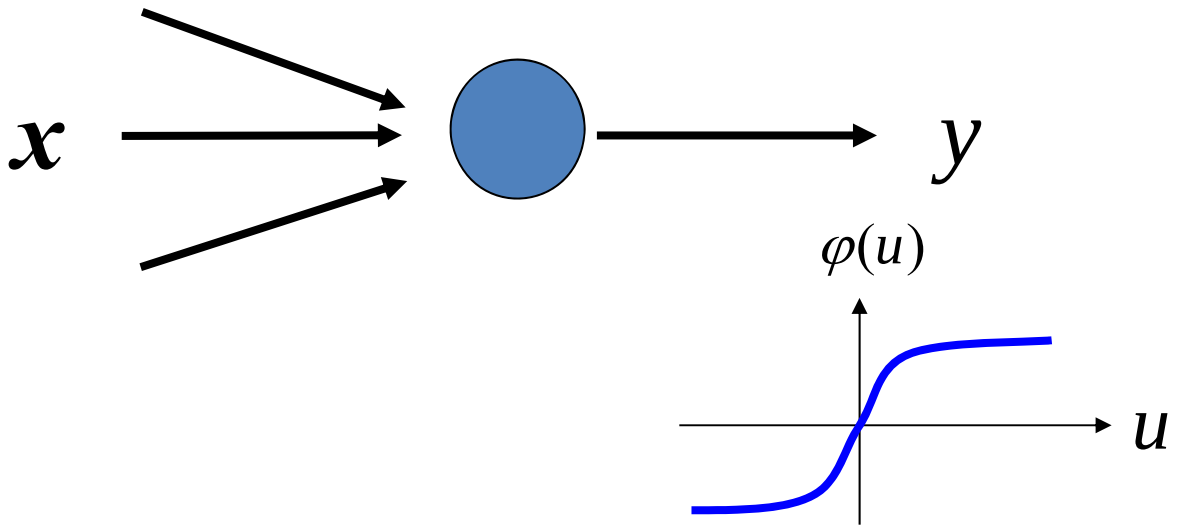
教師付学習：誤差逆伝搬

多層パーセプトロン
特異点!!

自然勾配学習：リーマン空間

数理ニューロン

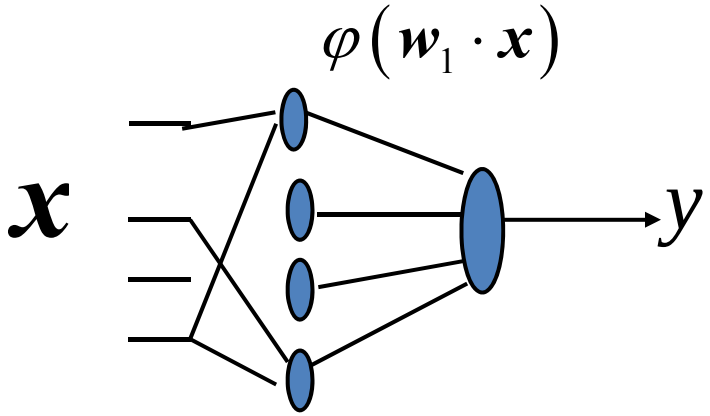
$$y = \varphi\left(\sum w_i x_i - h\right) = \varphi\left(\boldsymbol{w} \cdot \boldsymbol{x}\right)$$



多層パーセプトロン

$$y = \sum v_i \varphi(\boldsymbol{w}_i \cdot \boldsymbol{x})$$

$$x = (x_1, x_2, \ldots, x_n)$$



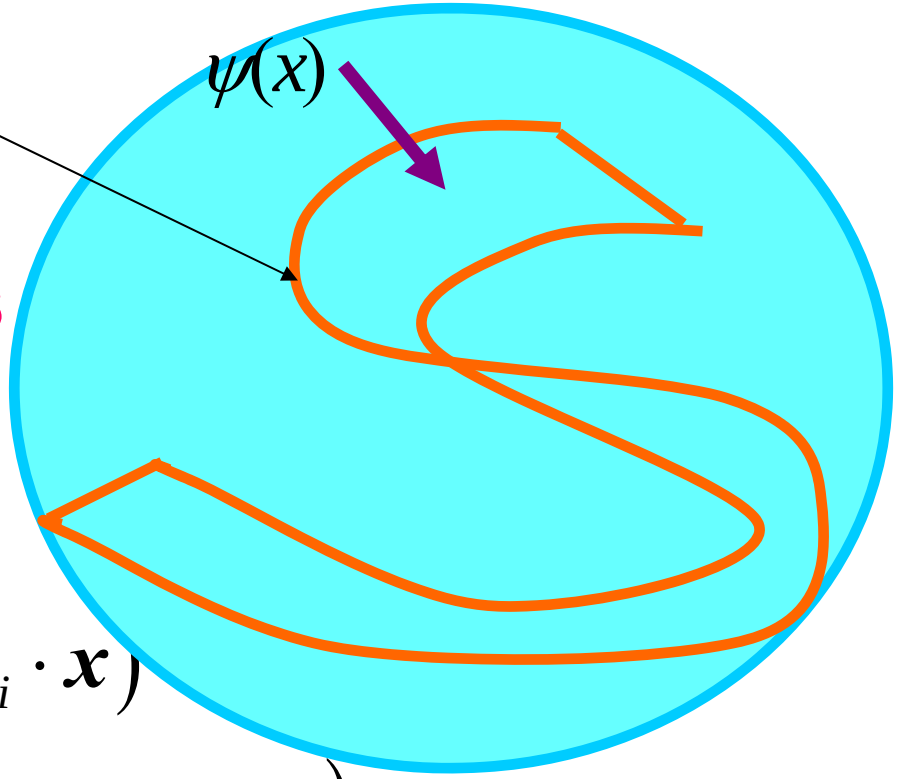
$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \sum v_i \varphi(\boldsymbol{w}_i \cdot \boldsymbol{x})$$

$$\theta = (w_1, \ldots, w_m; v_1, \ldots, v_m)$$

多層パーセプトロン

神経多様体

関数の空間 S



$$y = f(\boldsymbol{x}, \boldsymbol{\theta})$$

$$= \sum v_i \, \varphi \, (\boldsymbol{w}_i \cdot \boldsymbol{x})$$

$$\boldsymbol{\theta} = \left(v_1 \cdots, v_m \; ; \; \boldsymbol{w}_{1,} \cdots, \boldsymbol{w}_m \right)$$

確率降下学習 --- 誤差逆伝搬

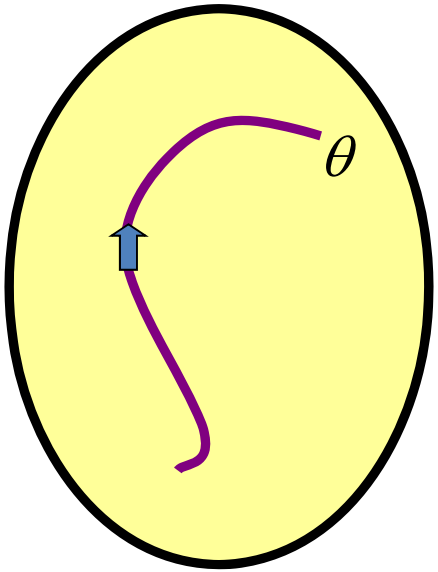
$$\text{examples}: (y_1, \boldsymbol{x}_1), \cdots (y_t, \boldsymbol{x}_t) \text{ --training set}$$

$$l(y, x; \theta) = \frac{1}{2} \left| y - f(\boldsymbol{x}, \boldsymbol{\theta}) \right|^2$$

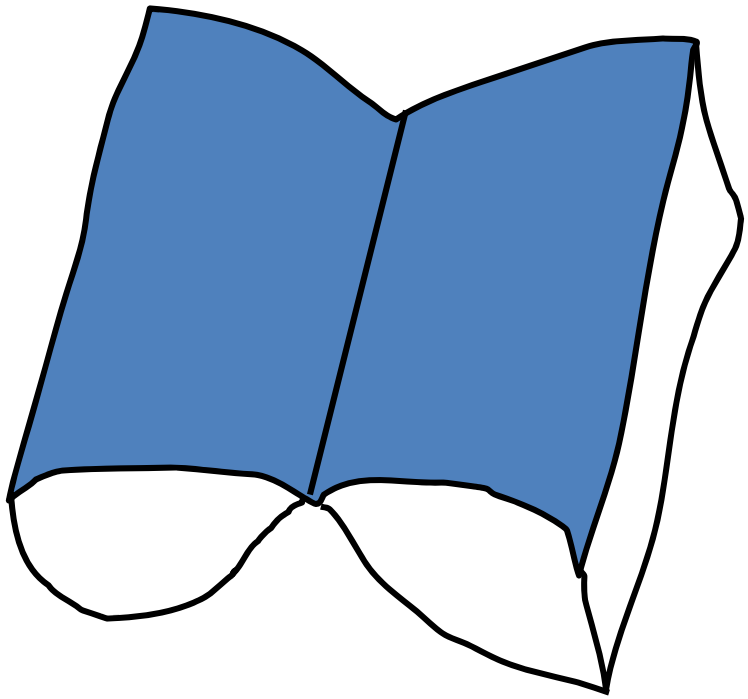
$$= -\log p(y, \boldsymbol{x}; \boldsymbol{\theta})$$

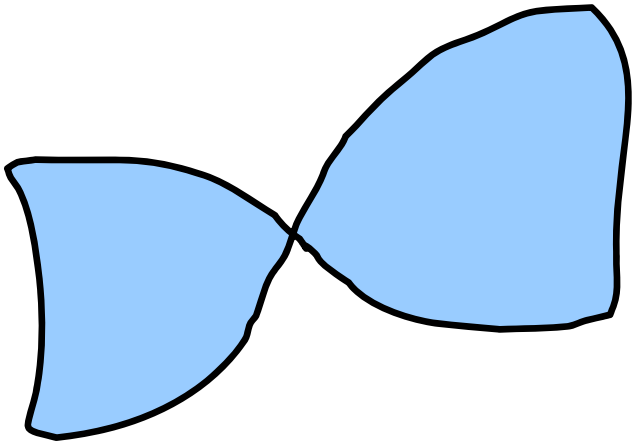
$$\Delta \boldsymbol{\theta}_t = -\eta_t \frac{\partial l(y_t, x_t; \theta_t)}{\partial \boldsymbol{\theta}}$$

$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \sum v_i \varphi(\boldsymbol{w}_i \cdot \boldsymbol{x})$$



特異点：特異構造

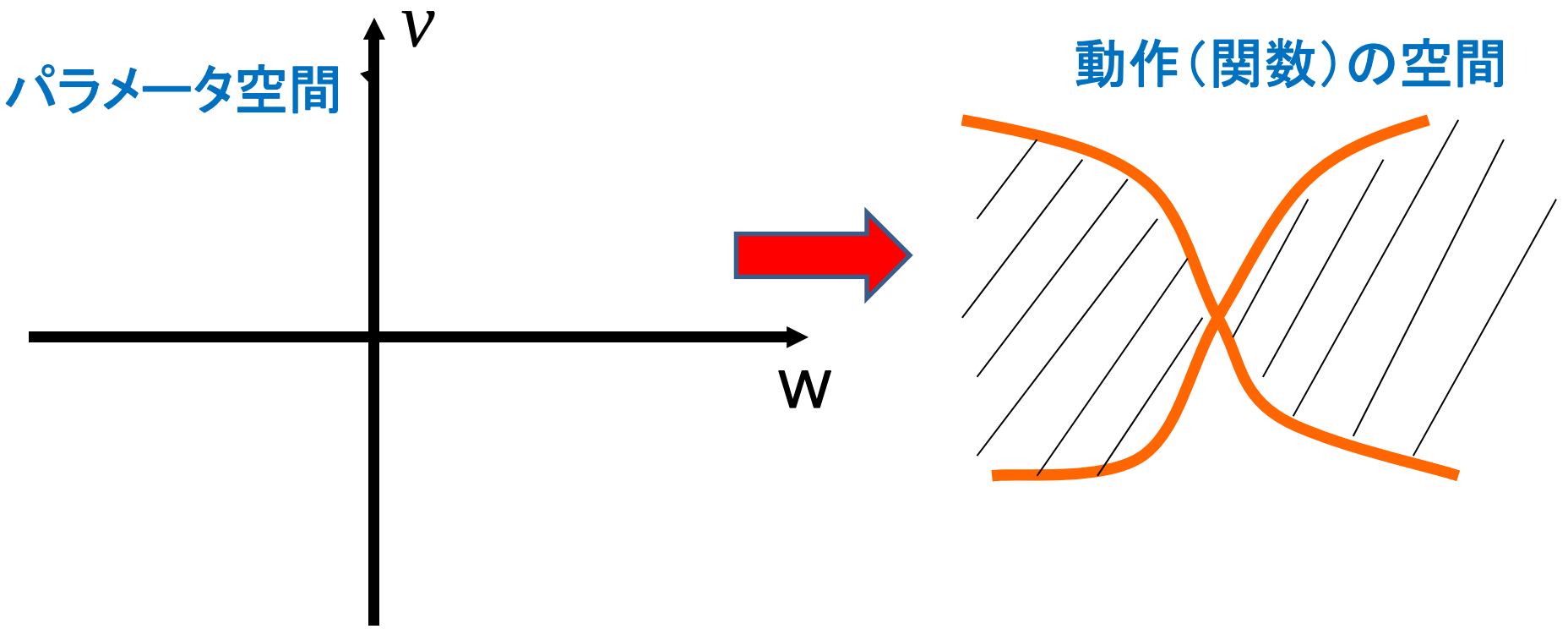




特異モデルの例

$$y = v\varphi\left(\boldsymbol{w}\cdot\boldsymbol{x}\right)+n$$

$$\mathrm{v}\,|\,\mathbf{w}\,|=0$$

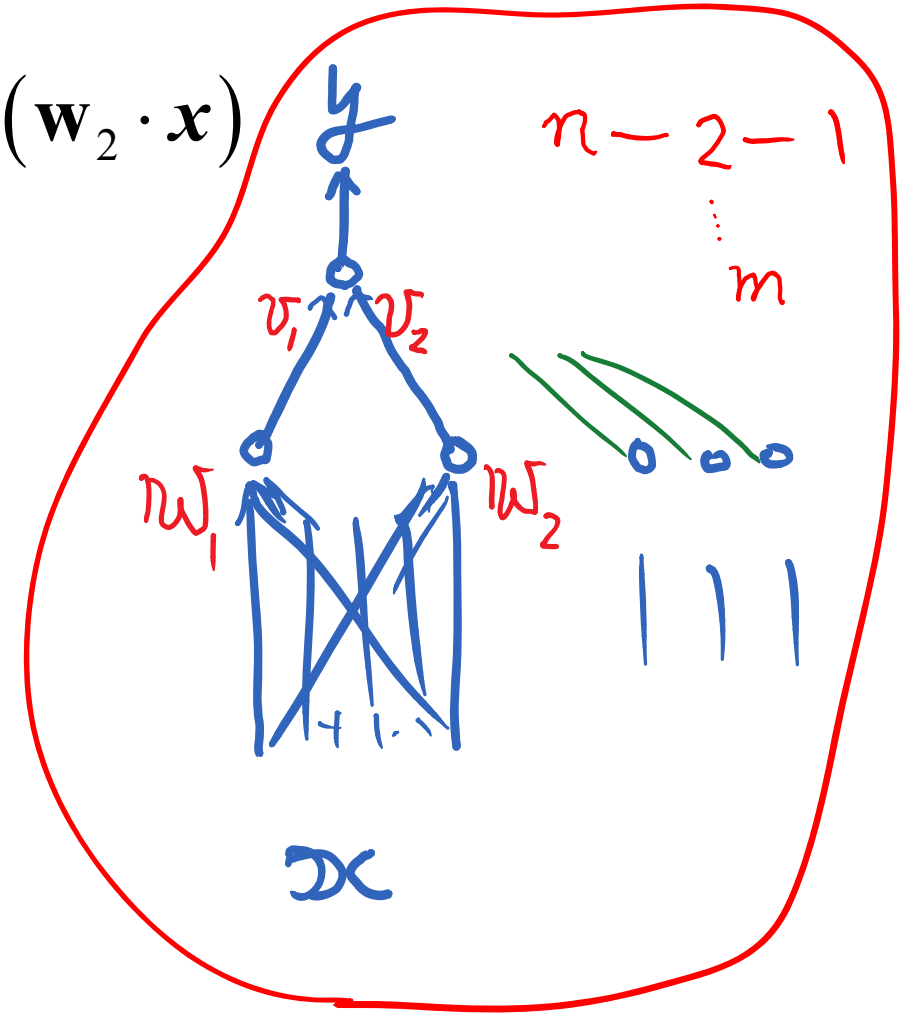


典型モデルの解析

$$f\left(\boldsymbol{x},\boldsymbol{\theta}\right)=v_1\varphi\left(\mathbf{w}_1\cdot\boldsymbol{x}\right)+v_2\varphi\left(\mathbf{w}_2\cdot\boldsymbol{x}\right)$$

$$y=f\left(\boldsymbol{x},\boldsymbol{\theta}\right)+\varepsilon$$

$$\varphi(u)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u}e^{-\frac{t^2}{2}}dt$$

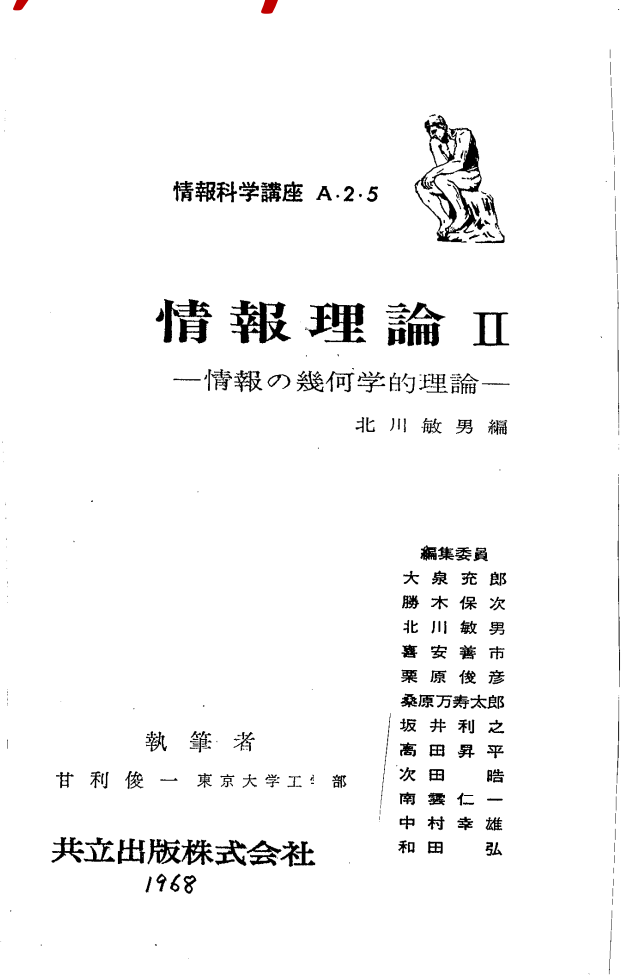


loss function: $l(\boldsymbol{x}, y; \boldsymbol{\theta}) = \frac{1}{2}\{y - f(\boldsymbol{x}, \boldsymbol{\theta})\}^2$

y : teacher signal : $\boldsymbol{\theta}_0$ **stochastic descent learning**

$\dot{\boldsymbol{\theta}} = -\eta \left\langle \frac{\partial l(\boldsymbol{x}_t, y_t, \boldsymbol{\theta}_t)}{\partial \boldsymbol{\theta}} \right\rangle$ **backprop : vanilla gradient**

First stochastic descent learning of MLP (1967;1968)


情報科学講座 A·2·5

情報理論 II

—情報の幾何学的理論—

北川敏男編

編集委員
大泉充郎
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共立出版株式会社

1968

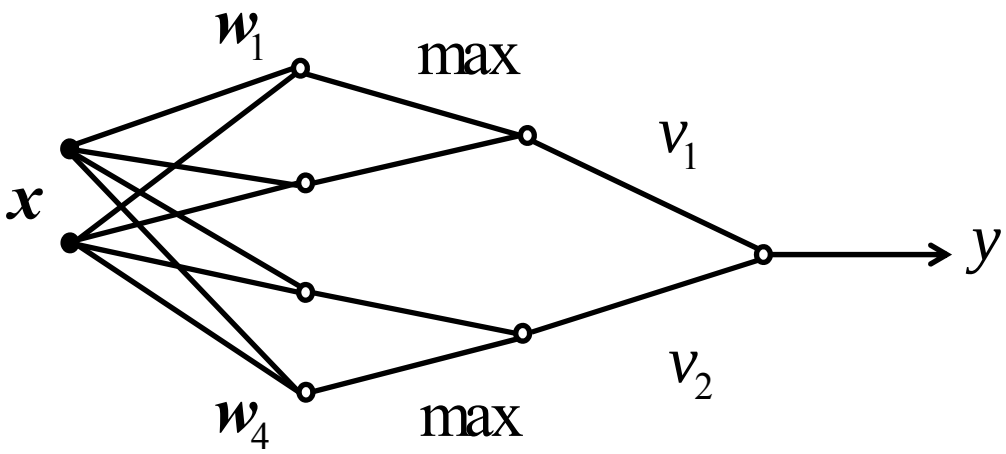
Information Theory II
--Geometrical Theory of Information

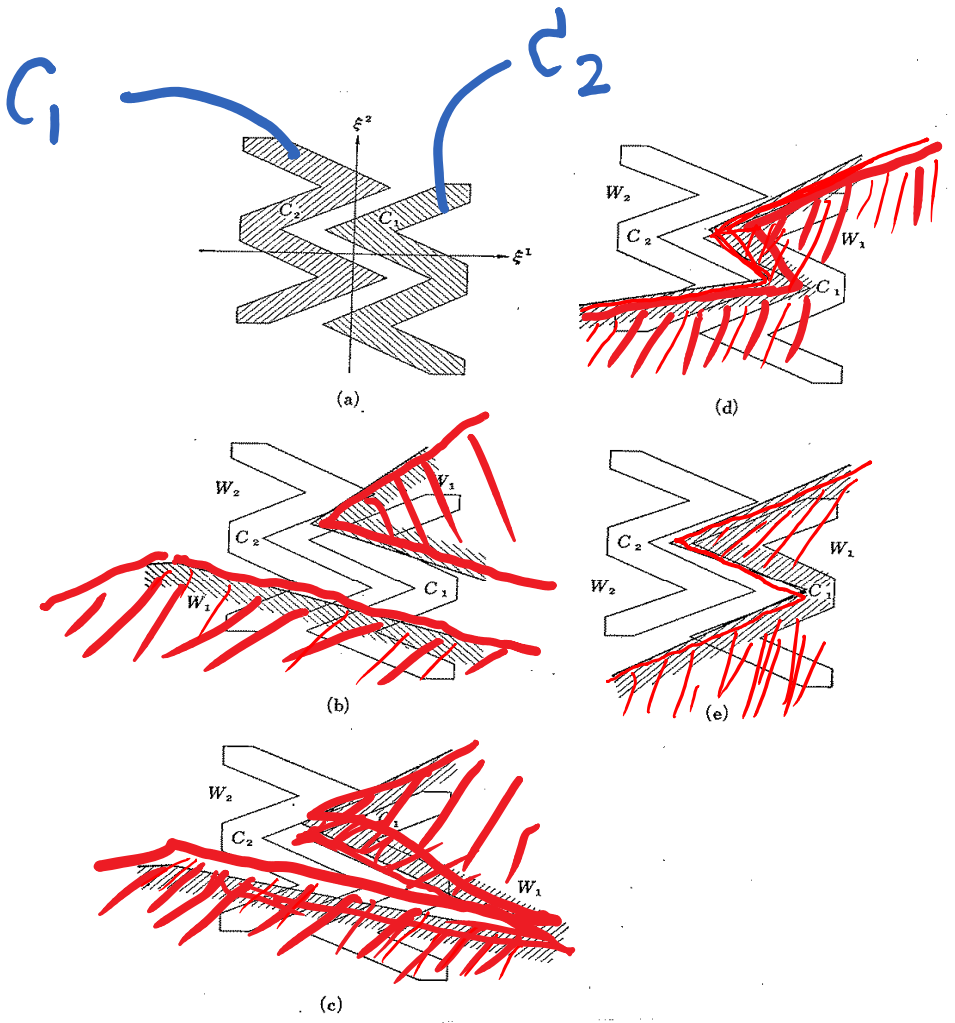
Shun-ichi Amari
University of Tokyo

Kyouritu Press, Tokyo, 1968

$$f\left(\boldsymbol{x},\boldsymbol{\theta}\right)=v_1\max\left\{\boldsymbol{w}_1\cdot\boldsymbol{x},\boldsymbol{w}_2\cdot\boldsymbol{x}\right\}+v_2\max\left\{\boldsymbol{w}_3\cdot\boldsymbol{x},\boldsymbol{w}_4\cdot\boldsymbol{x}\right\}$$

$$v_1=1;\ v_2=-1$$




C1
C2
ξ²
ξ¹
C₂
C₁
(a)
W₂
C₂
W₁
C₁
(d)
W₂
W₁
C₂
C₁
W₁
(b)
C₂
W₁
W₂
C₁
(e)
W₂
C₂
W₁
(c)

自然勾配学習法

$$\dot{\boldsymbol{\theta}} = -\eta G^{-1}(\boldsymbol{\theta}_t)\left\langle \nabla_{\theta}(\boldsymbol{x}_t, y_t, \boldsymbol{\theta}_t)\right\rangle$$

$$\nabla_{\theta} = \frac{\partial}{\partial \boldsymbol{\theta}}$$

$G(\boldsymbol{\theta}) = \left\langle \nabla_{\theta} l \nabla_{\theta} l \right\rangle$ ： **Fisher Information Matrix**

不変； 最急降下

自然勾配 (Riemannian)

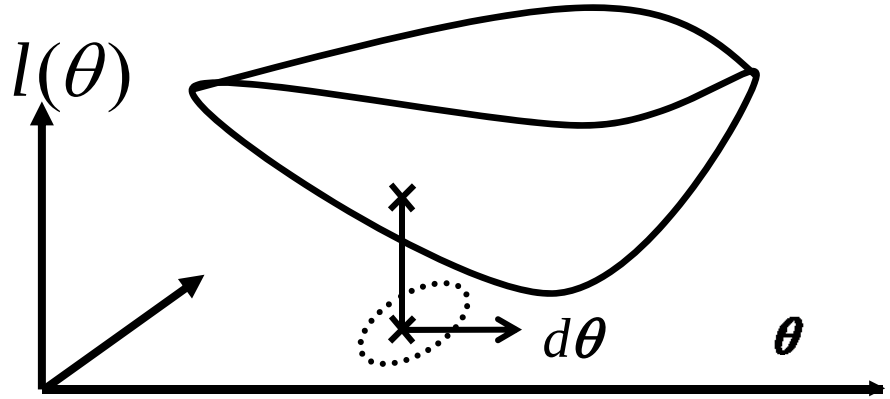
$$\max \qquad dl = l(\boldsymbol{\theta} + d\boldsymbol{\theta}) - l(\boldsymbol{\theta})$$

$$\left|d\boldsymbol{\theta}\right|^2 = \varepsilon$$

$$\tilde{\nabla} l = G^{-1}(\boldsymbol{\theta}) \nabla l$$

$$\Delta \boldsymbol{\theta}_t = -\eta_t \tilde{\nabla} l(x_t, y_t; \theta_t)$$

最急降下---Natural Gradient



$$\nabla l = \left(\frac{\partial l}{\partial \theta_1}, \cdots, \frac{\partial l}{\partial \theta_n} \right)$$

$$\Delta \boldsymbol{\theta}_t = -\eta_t \nabla l(x_t, y_t; \theta_t)$$

$$\widetilde{\nabla} l = G^{-1}(\boldsymbol{\theta}) \nabla l$$

$$\left| d\boldsymbol{\theta} \right|^2 = d\boldsymbol{\theta}^T G d\boldsymbol{\theta} = \sum G_{ij} d\theta^i d\theta^j$$

自然勾配の長所

Steepest descent; invariant　　Yan Ollivier

Fisher-efficient

Natural gradient is non-vanishing even in multiple layers

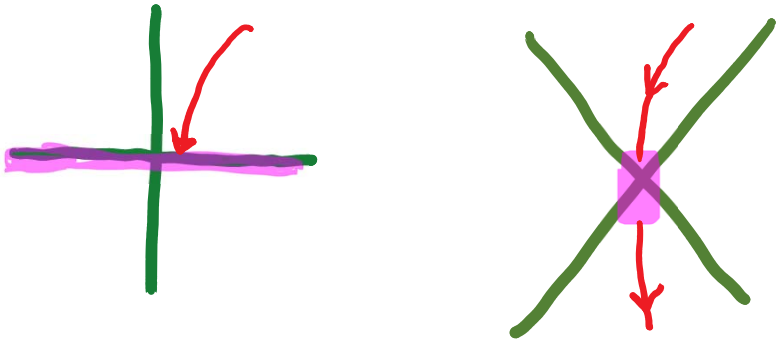
Good at singular regions　(avoid plateaus: Milnor attractor)

適応自然勾配

$$G_{t+1}^{-1} = (1+\varepsilon) G_t^{-1} - \varepsilon G_t^{-1} \nabla l(x_t) \nabla l(x_t)^\top G_t^{-1}$$

$G^{-1} \to \infty$, $\nabla l \to 0$ at singularities

$G^{-1} \nabla l$



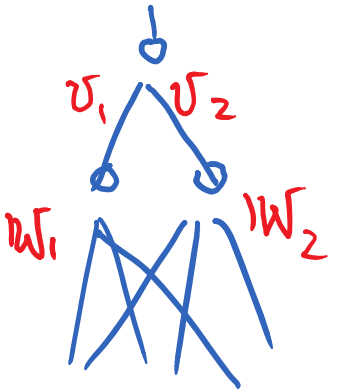
特異領域

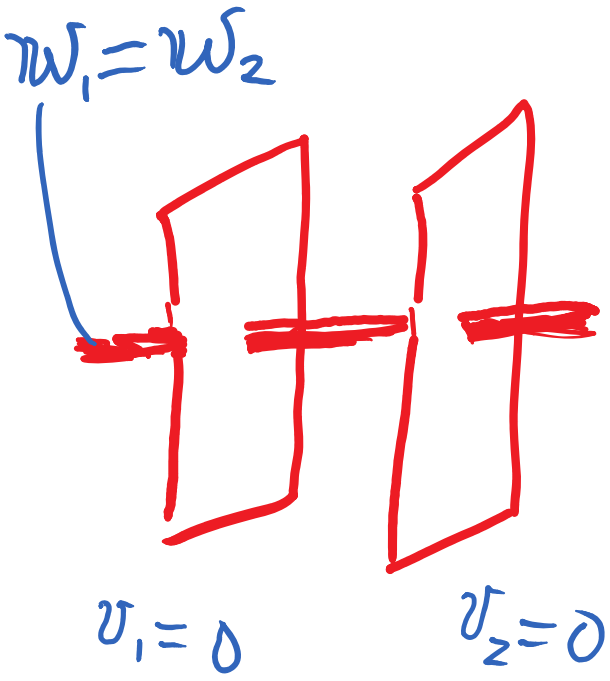
$$R(\mathrm{v},\mathbf{w})=\left\{\boldsymbol{\theta}\middle|\mathbf{w}_1=\mathbf{w}_2=\mathbf{w},v_1+v_2=v\right\}$$

$$\cup\left\{\boldsymbol{\theta}\middle|v_1=0,v_2=v,\mathbf{w}_2=\mathbf{w}\right\}$$

$$\cup\left\{\boldsymbol{\theta}\middle|v_1=v,\mathrm{v}_2=0,\mathbf{w}_1=\mathbf{w}\right\}$$

$$f\left(\boldsymbol{x},\boldsymbol{\theta}\right)=w_1\varphi\left(\boldsymbol{J}_1\cdot\boldsymbol{x}\right)+w_2\varphi\left(\boldsymbol{J}_2\cdot\boldsymbol{x}\right)$$





座標変換

$$\boldsymbol{v} = \frac{w_1 \boldsymbol{J}_1 + w_2 \boldsymbol{J}_2}{w_1 + w_2},$$

$$w = w_1 + w_2,$$

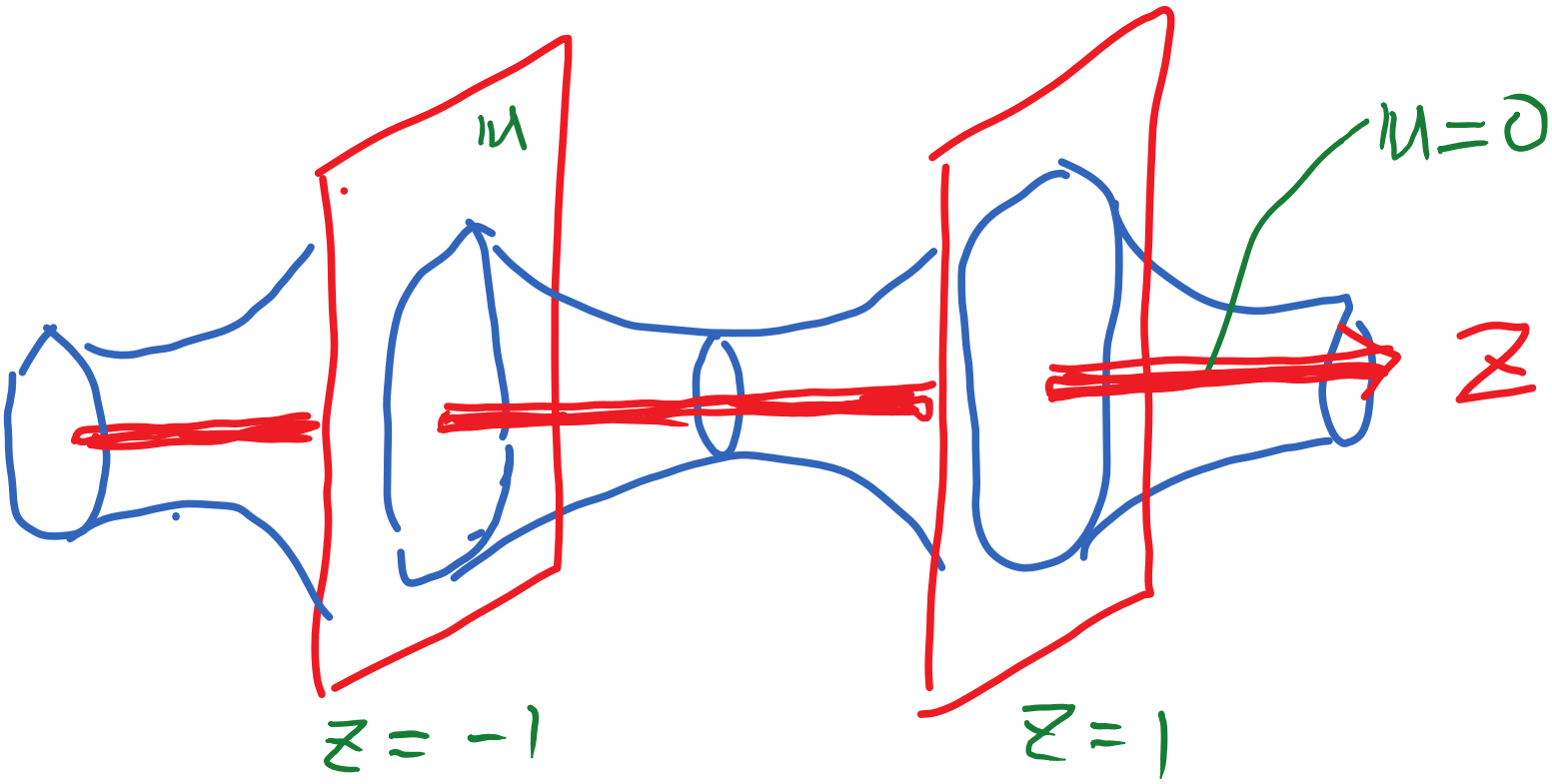
$$\boldsymbol{u} = \boldsymbol{J}_2 - \boldsymbol{J}_1,$$

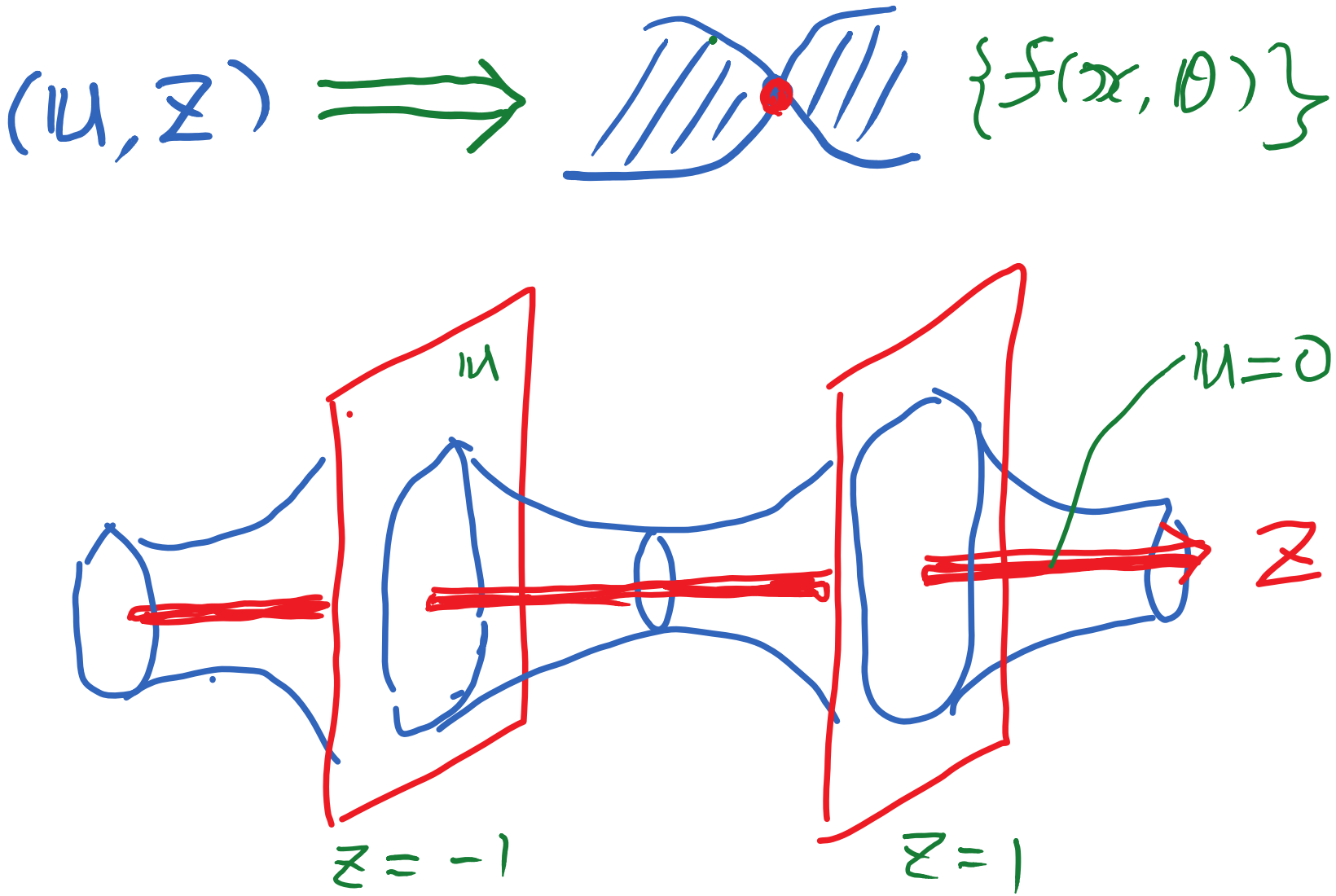
$$z = \frac{w_2 - w_1}{w_1 + w_2}$$

$$\boldsymbol{\xi} = \left(\boldsymbol{v}, w, \boldsymbol{u}, z\right)$$

特異領域

$$R(\mathrm{v},\mathbf{w}) = \{\boldsymbol{u} = 0\} \cup \{z = \pm 1\}$$




(u, z)
{f(x, θ)}
u
u=0
z
z = -1
z = 1

Taylor expansion $\boldsymbol{u}$: small

$$f\left(\boldsymbol{x},\boldsymbol{\xi}\right)=w\varphi\left(\boldsymbol{v}\cdot\boldsymbol{x}\right)+\frac{w}{8}\varphi''\left(\boldsymbol{v}\cdot\boldsymbol{x}\right)\left(1-z^2\right)\left(\boldsymbol{u}\cdot\boldsymbol{x}\right)^2$$

$$-\frac{w}{24}\varphi'''\left(\boldsymbol{v}\cdot\boldsymbol{x}\right)z\left(1-z^2\right)\left(\boldsymbol{u}\cdot\boldsymbol{x}\right)^3+\cdots$$

fast dynamics $\rightarrow w,\boldsymbol{v}$: stability

slow dynamics $\rightarrow u,\boldsymbol{z}$

Rの近傍でのダイナミックス

$$\dot{\boldsymbol{u}} = -\frac{\eta w^*}{2}\left(1-z^2\right)\left\langle \varphi'' e\left(\boldsymbol{u}\cdot\boldsymbol{x}\right)\boldsymbol{x}\right\rangle$$

$$\dot{z} = -\frac{\eta}{4w^*} z\left(3+z^2\right)\left\langle \varphi'' e\left(\boldsymbol{u}\cdot\boldsymbol{x}\right)^2\right\rangle$$

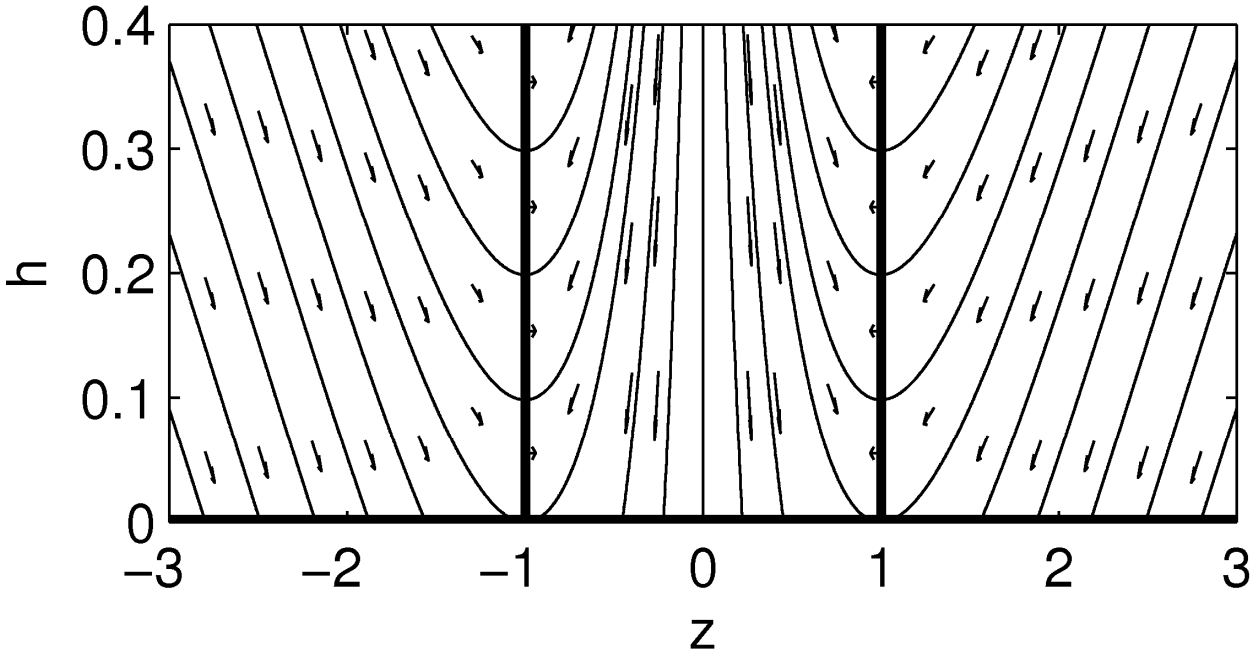
解の軌道

$$\frac{1}{2}\left|\boldsymbol{u}(t)\right|^2 = \frac{2}{3} w^* \log\frac{\left(z^2+3\right)^2}{\left|z(t)\right|} + c$$

安定論 1

true solution is in R:

$$R = \{ \boldsymbol{u} = 0 \ \text{ or } \ z = \pm 1 \} \ : \text{stable}$$



Dynamic vector fields: Redundant case

安定論 2： true solution is outside R

$$H = \left\langle \varphi'' e(\boldsymbol{x}) \boldsymbol{x}\boldsymbol{x}^T \right\rangle$$

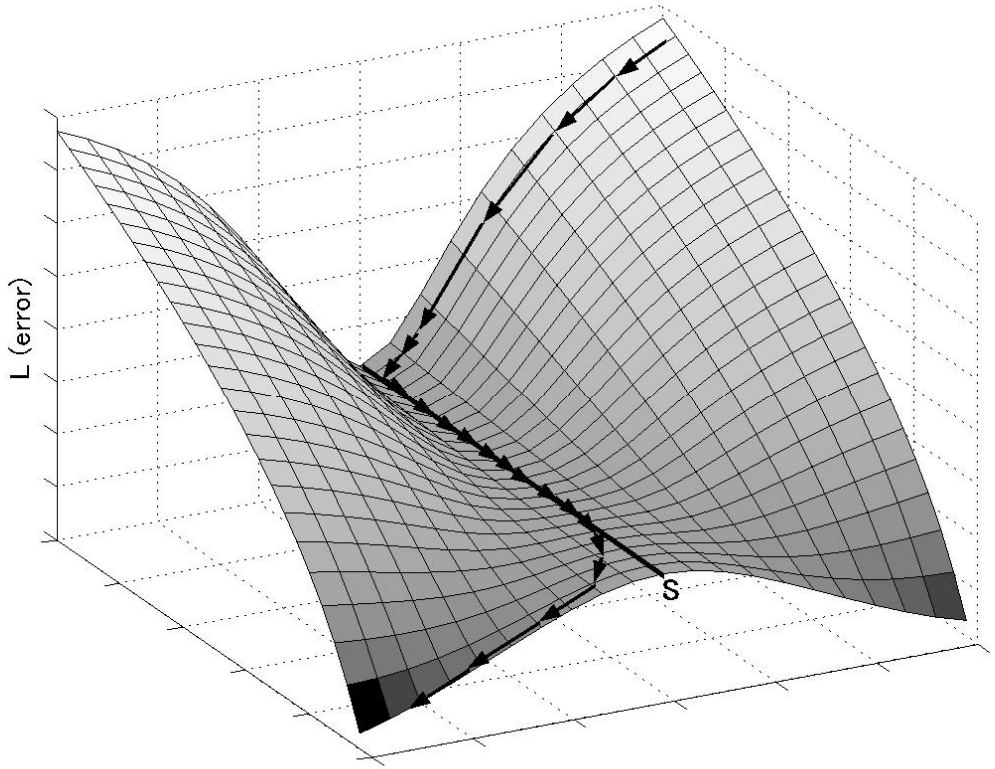
wH : positive-definite

$\Leftrightarrow$ $|z| < 1$ stable ; $|z| > 1$ unstable

wH : negative-definite

$\Leftrightarrow$ $|z| > 1$ stable ; $|z| < 1$ unstable

特異領域における解の軌道



Milnor attractor

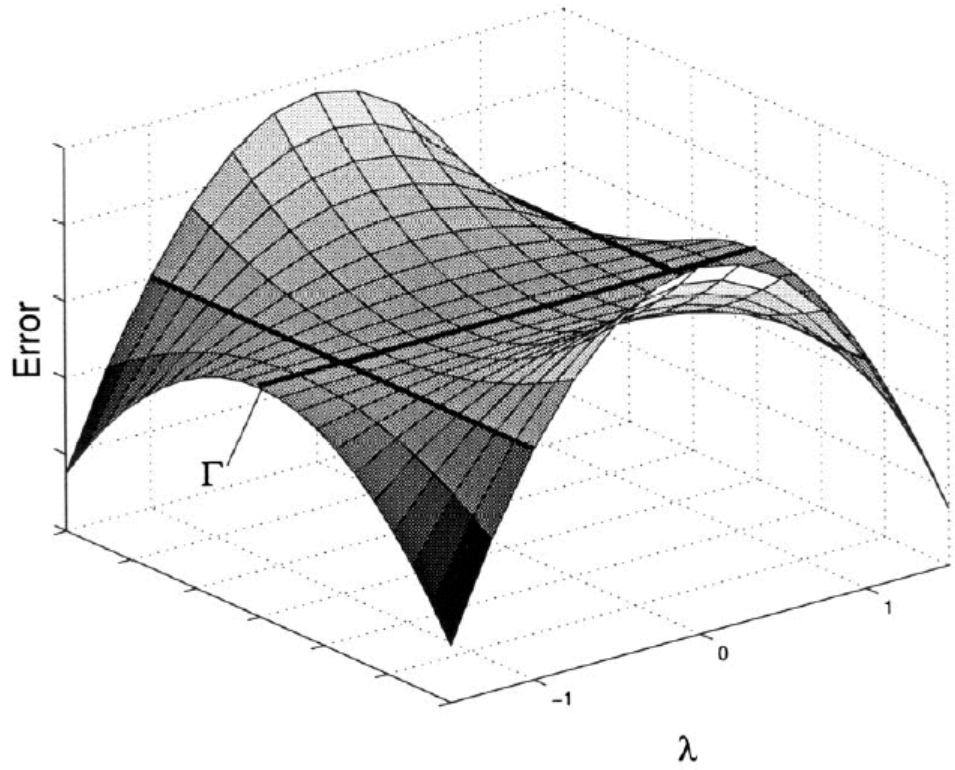


Fig. 5. Critical set with local minima and plateaus.

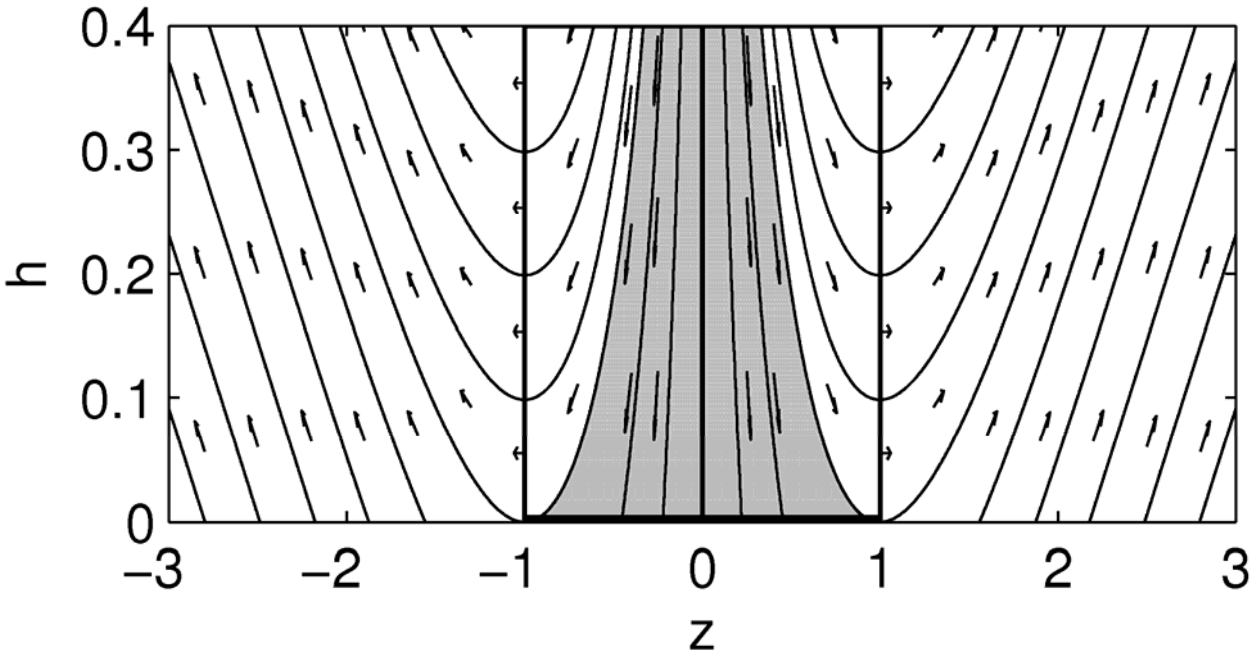
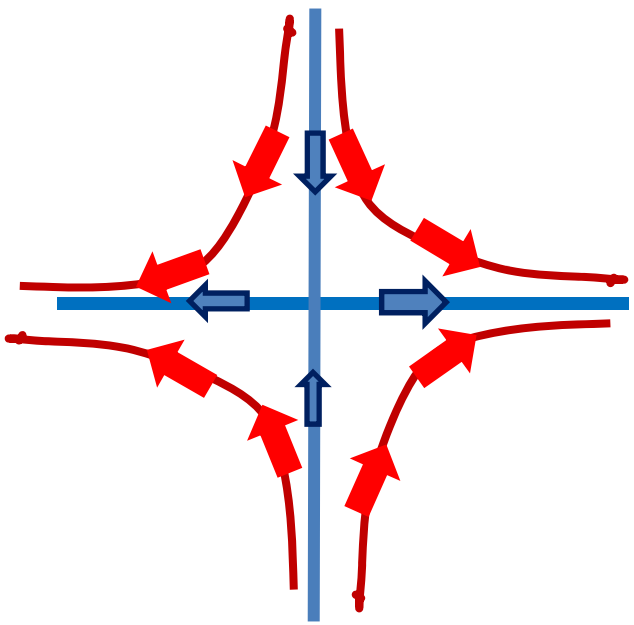
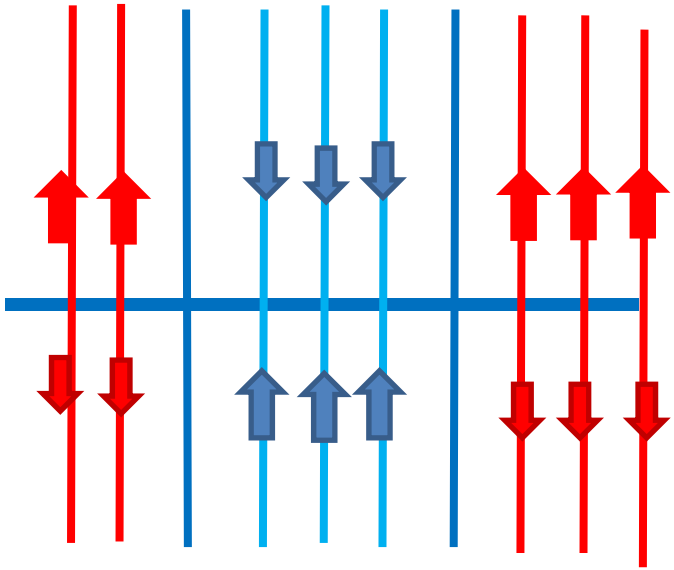


Fig. 2: trajectories

鞍点とプラトー





学習の遅滞

$$E = \frac{1}{2}\left\langle e^2 \right\rangle$$

$$\dot{E} = O\left(u^5\right)$$

$$\dot{E} = O\left(u^2\right)$$

特異領域Rのトポロジー

blow-down coordinates ： $\xi=(\mathbf{u},z)\Rightarrow\boldsymbol{\alpha}=(\tau,\sigma,\boldsymbol{e})$

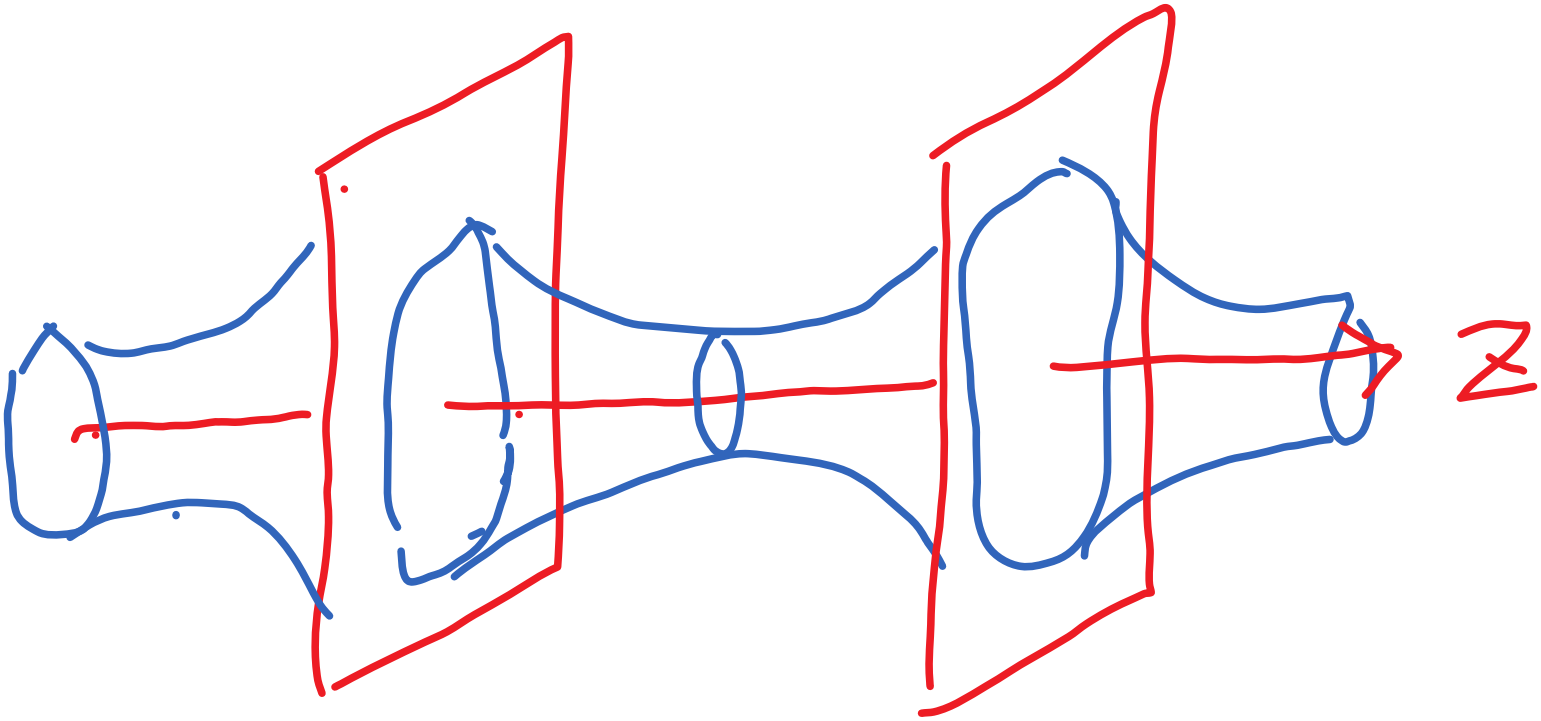
$$\tau=c_1\left(1-z^2\right)u^2, \qquad u=|\boldsymbol{u}|$$

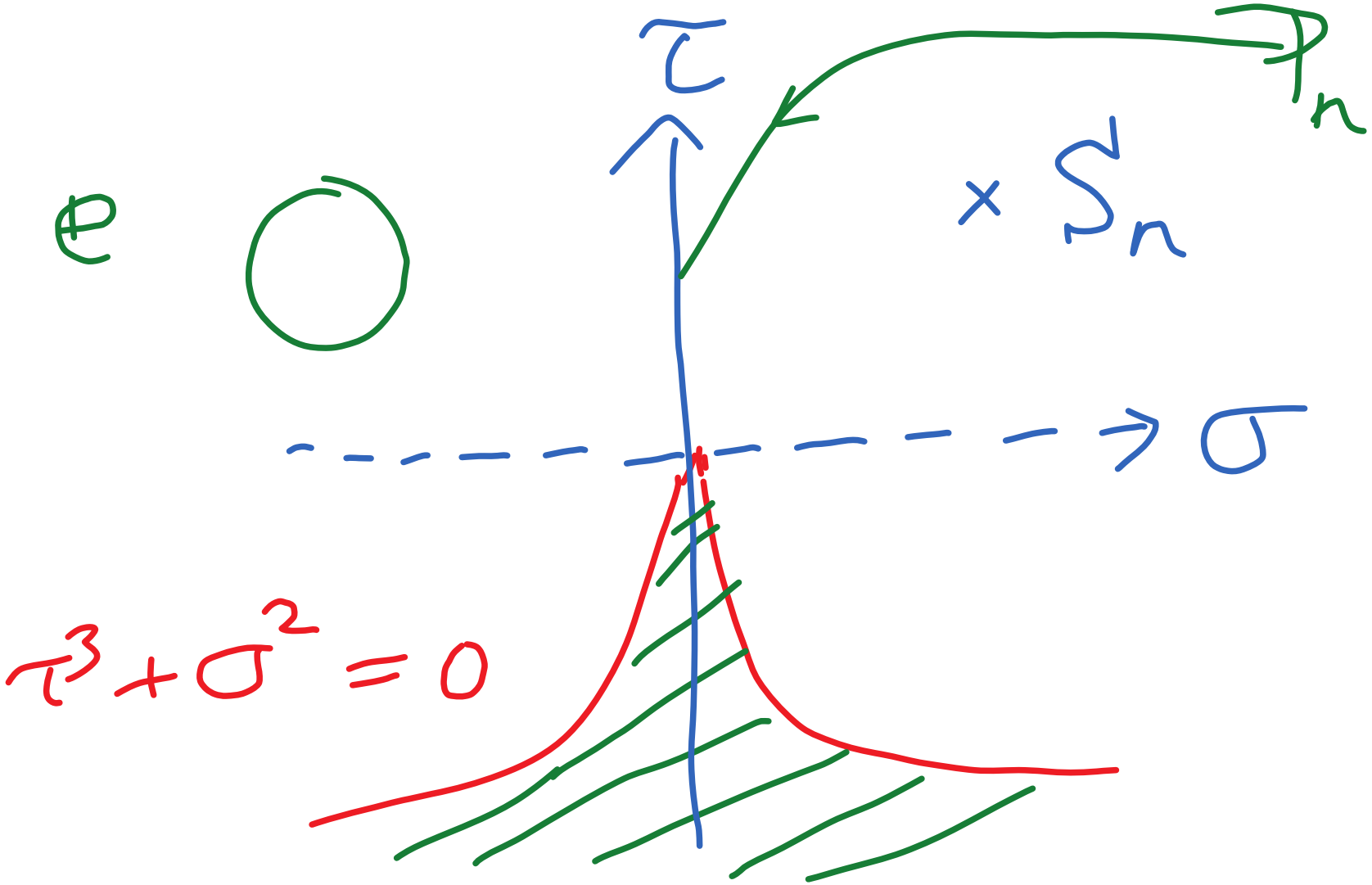
$$\sigma=c_2 z\left(1-z^2\right)u^3,$$

$$\boldsymbol{e}=\frac{\boldsymbol{u}}{|\boldsymbol{u}|}\in S_n, \qquad |\boldsymbol{e}|=1$$

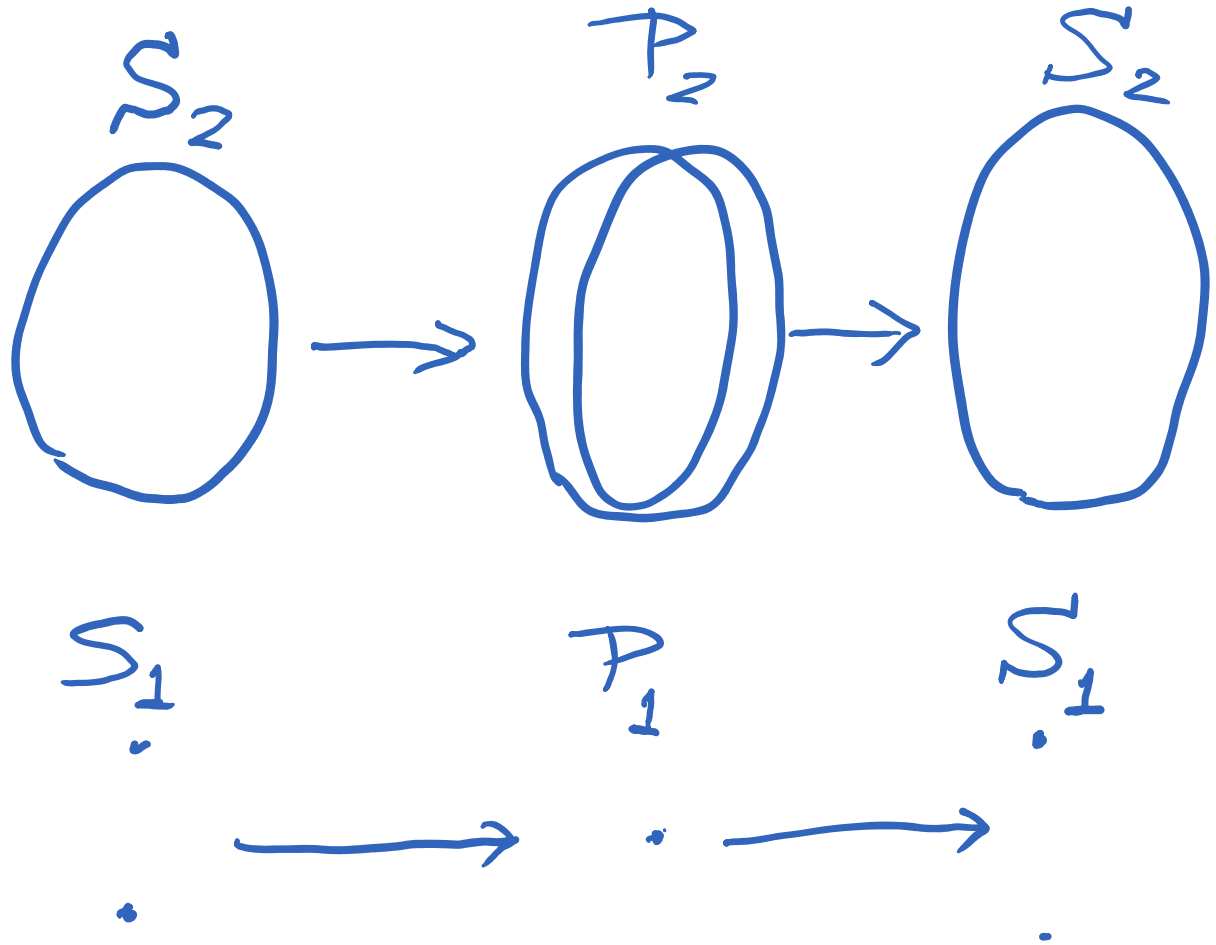
Singular Region

$$R(v,\mathbf{w}) = \{\boldsymbol{u} = 0\} \bigcup \{z = \pm 1\}$$




τ
γ_n
$\times S_n$
σ
$\tau^3+\sigma^2=0$

Sphere *Sn* and Projective space *Pn*



特異点における自然勾配学習

$$\frac{d}{dt}\begin{pmatrix}\tau\\ \sigma\end{pmatrix} = -\eta\begin{pmatrix}\tau\\ \sigma\end{pmatrix} \qquad : \quad \text{true model} \in R$$

$$\frac{d}{dt}\begin{pmatrix}\tau\\ \sigma\end{pmatrix} = O(1) \qquad : \quad \text{true model} \notin R$$

Milnor attractor

自然勾配学習の近似実現法

adaptive natural gradient

$$G_{t+1}^{-1} = (1+\varepsilon)G_t^{-1} - \varepsilon G_t^{-1}\nabla l \nabla l G_t^{-1}$$

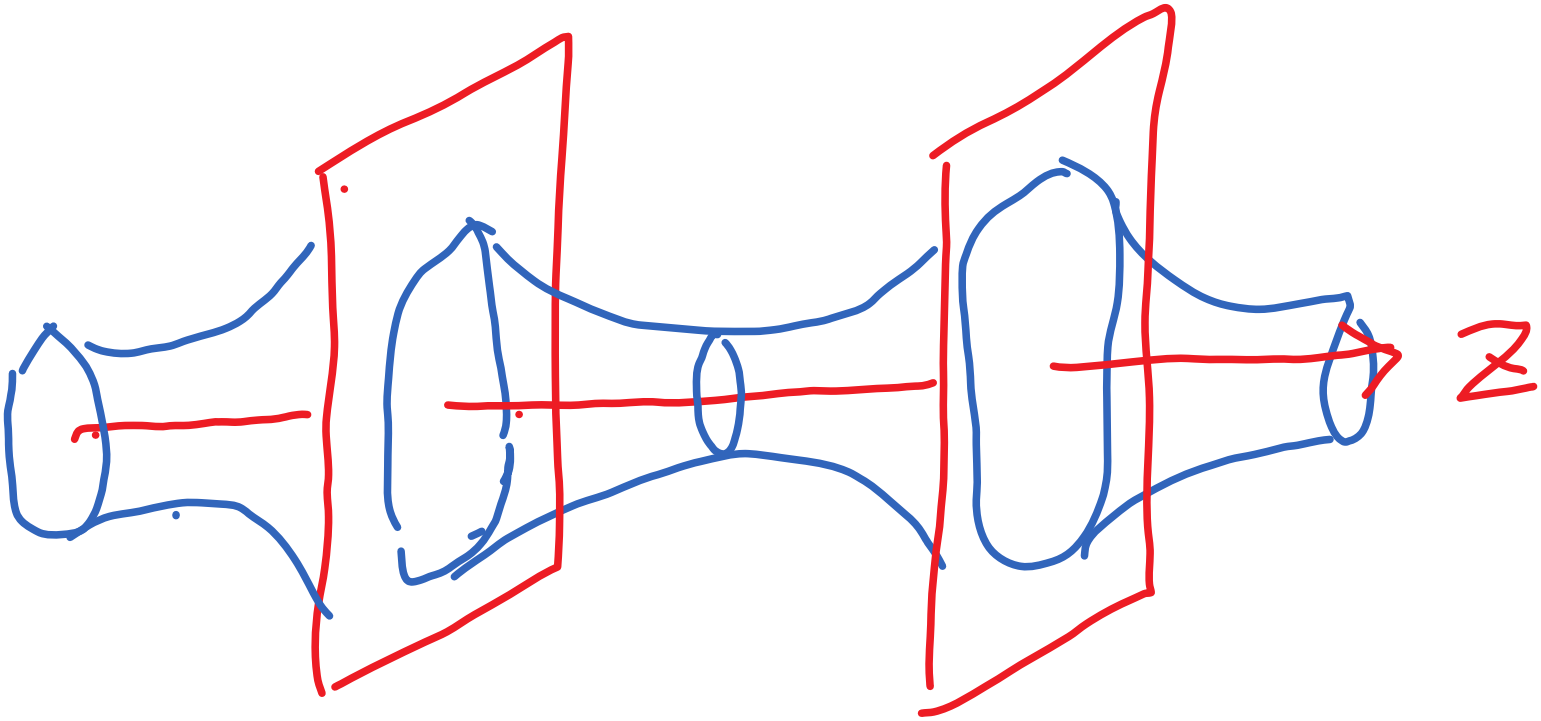
Unitwise diagonalization of G : Yan Olliver

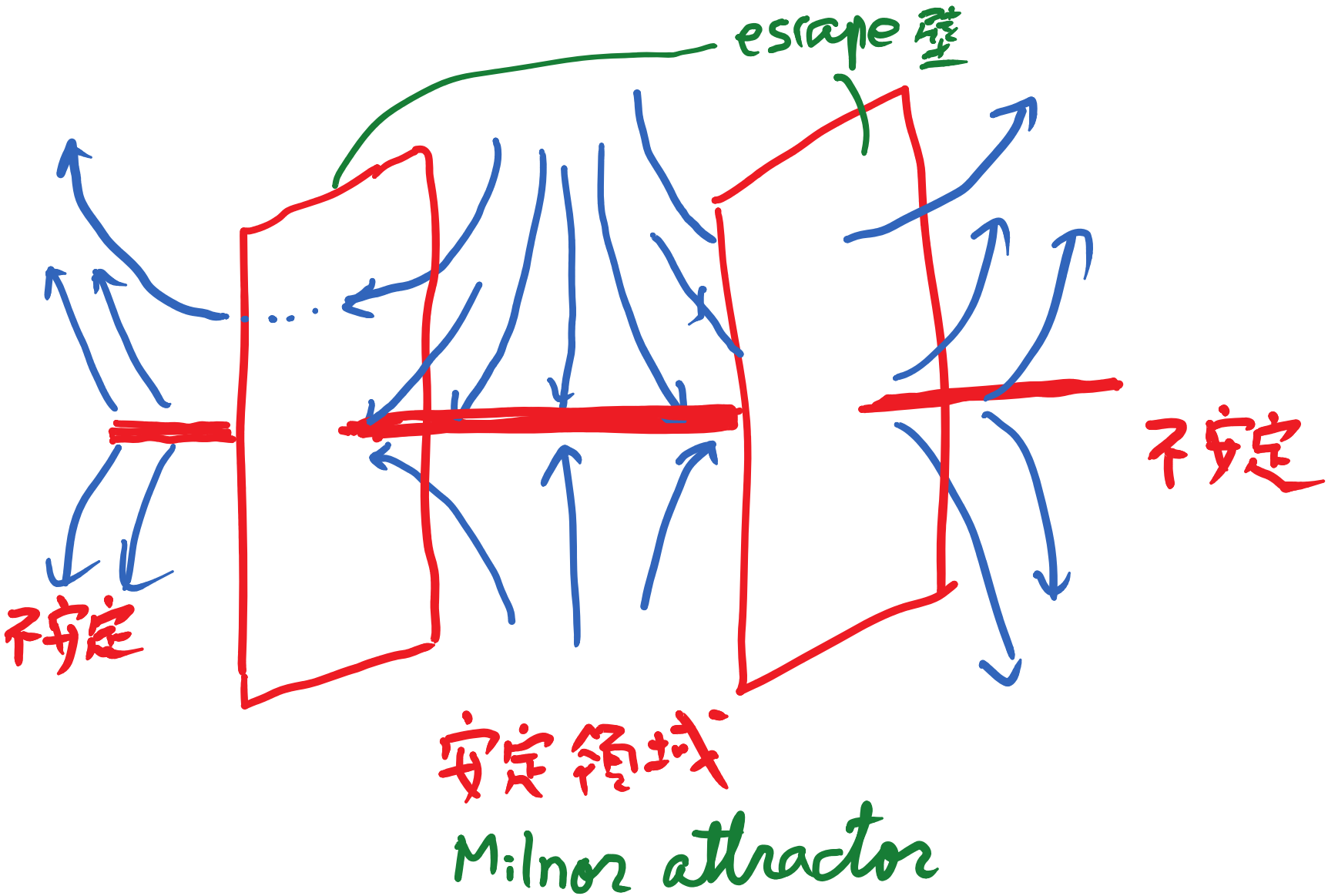
$G^{-1}\nabla l$: non-singular

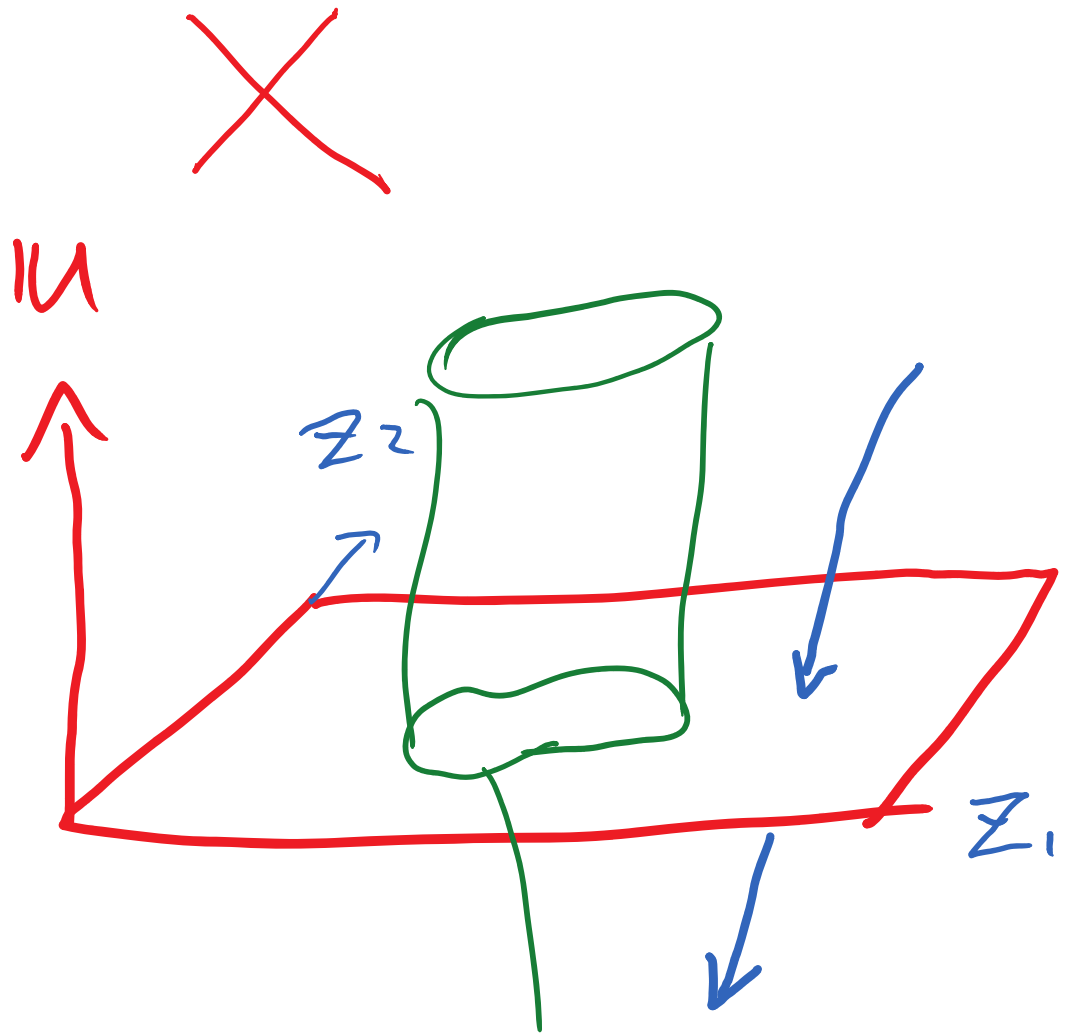
G: unitwise-diagonalization is OK (Ollivier)

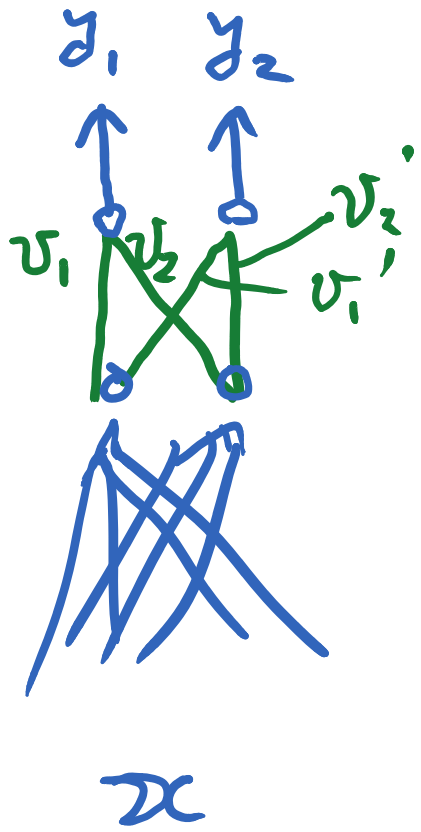
Singular Region

$$R(v,\mathbf{w}) = \{\boldsymbol{u} = 0\} \bigcup \{z = \pm 1\}$$




escape 壁
不安定
不安定
安定領域
Milnor attractor





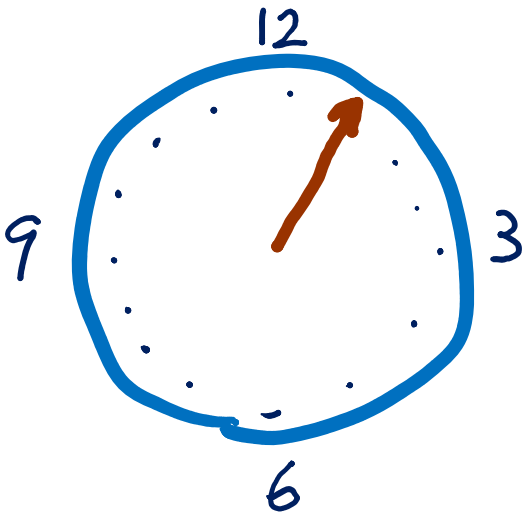
安定領域の消失：通過点

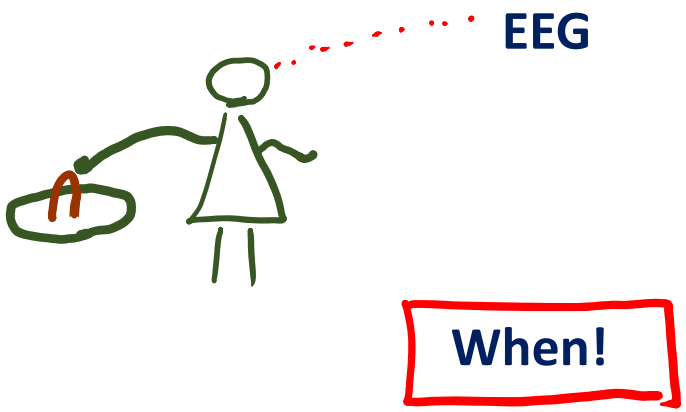
脳に何を学ぶのか
意識と無意識のダイナミックス

記号 --- 興奮パターン
論理的推論 --- 並列ダイナミックス

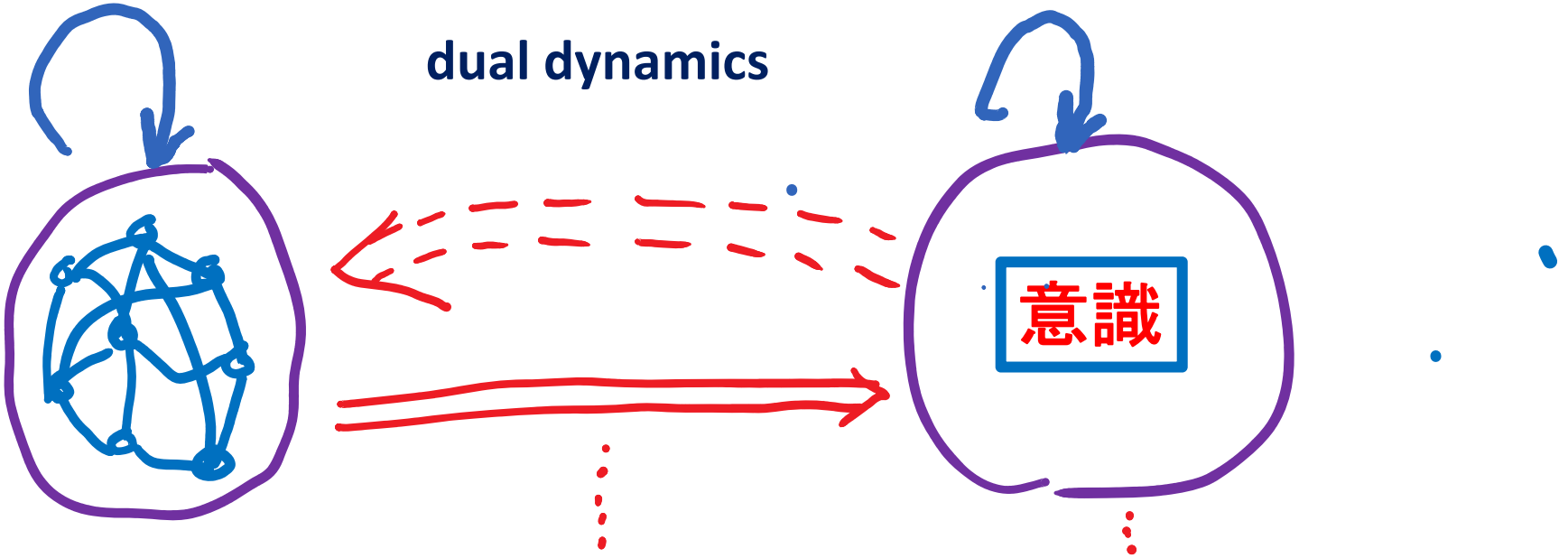
AI NN

Libet の実験：自由意志





予測（先付け）と後付け　Prediction and Postdiction



ダイナミックス　　意思決定と行動　　反省、正当化、論理

意識の発生

共同作業、自分の意図を自分で知る

言語：　論理的思考、数学

AIと脳科学の将来

Postdiction（後付け）：　記号と論理を駆使
→パターン力学への介入

脳型の記憶方式　（連想記憶）

原理の共有；存在証明

意識の役割

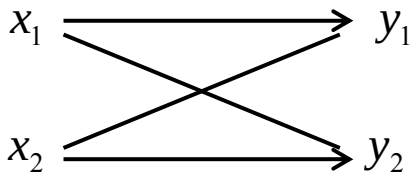
情報統合理論と情報幾何：意識の定量化

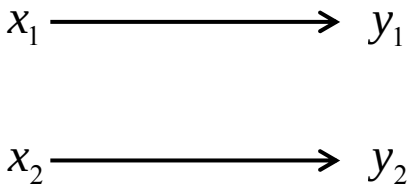
多層パーセプトロンのトポロジー

規範ダイバージェンスは定義できるか

多端子統計推論

甘利俊一　理化学研究所脳科学総合研究センター





full モデル: $S_F = \{p(\mathbf{x},\mathbf{y})\}$

分離モデル: $S_S = \{q(\mathbf{x},\mathbf{y})\}$

$$q(\mathbf{y} \mid \mathbf{x}) = \Pi q(y_i \mid x_i)$$

Tononi, Barrett and Seth, Ay

Split Model S_G

$$q(\boldsymbol{x},\boldsymbol{y}) = q_X(\boldsymbol{x})\tilde{q}_Y(\boldsymbol{y})\prod q(y_i|x_i)$$

$$\theta_{12}^{XY} = \theta_{21}^{XY} = 0$$

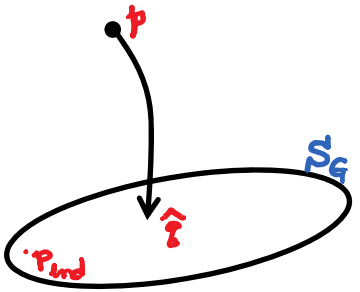
$$q(x_1, y_2|x_2, y_1) = q(x_1|x_2, y_1)\,q(y_2|x_2, y_1)$$

$$0 \le \Phi \le I(X:Y)$$

$$\hat{q}_X(\boldsymbol{x}) = p_Y(\boldsymbol{x}), \qquad \hat{q}_Y(\boldsymbol{y}) = p_Y(\boldsymbol{y})$$

$$\hat{q}(y_i|x_i) = p(y_i|x_i)$$

graphical model



要請 1: $\Phi = D[p:q] \qquad q:\text{split}$

要請 2: Markov条件を満たす最小の split モデル

$$X_1 \to X_2 \to Y_2$$
$$X_2 \to X_1 \to Y_1$$

要請 3 D は KL-ダイバージェンス

$S_H \subset S_G, S_D$: S_G, S_H dually flat

$S_M \subset S_G$: S_M, S_D not flat

$\Phi_G \leq \Phi_H, \Phi^*$; $\Phi_D \leq \Phi_H$