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Extending the Use of Instrumental Variables for the Identification of Direct Causal Effects in SEMs

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Abstract: In this paper, we present an extended set of graphical criteria for the identification of direct causal effects in linear Structural Equation Models (SEMs). Previous methods of graphical identification of direct causal effects in linear SEMs include methods such as the single door criterion, the instrumental variable and the IV-pair, and the accessory set. However, there remain graphical models where a direct causal effect can be identified and these graphical criteria all fail. We present a few of these examples, and presents an extended set of graphical criteria which uses descendants of the cause variable as "path-specific instrumental variables". The results can be used to identify the direct causal effect as long as an certain conditions based on an extended set of graphical criteria and the identifiability of other causal effects are satisfied.

Keywords: Structural Equation Models, Causality, Parameter Identification

1 Introduction

Structural Equation Models (SEM) is a useful tool for causal analysis, and is widely used in areas of social science such as economics [Bollen 1989, Duncan 1975]. Research by scientists, social scientists, and computer scientists in this area has allowed the problem to be applied in real-life models.

In a linear SEM, the relationships between observed variables are expressed in linear equations. The structure of the equations is such that they not only express the linear relationships between the variables, together with a stochastic error term for unobserved factors, but also the causal dependence among the observed variables. For each variable Y, its structural equation where it appears on the left-hand side, the presence (and absence) of a variable X on the right-hand side specifies that X is (or is not) a direct cause of Y.

A fundamental problem in linear SEMs is to estimate the strength of a certain direct causal effect from one variable to another from a combination of observed data and model structure. This is called the *identification problem* [Fisher 1966]. Although many methods, both algebraic and graphical, have been developed over the years, this problem is still not solved. Currently there are no sufficient or necessary criteria for deciding whether a causal effect can be identified from observed data. Current identification methods based on graphical criteria, including the single door criterion, the front door criterion and the back door criterion, the instrumental variable and the IV-pair, and the accessory set, can be used to identify direct causal effects only when certain conditions are met. However, there exist direct causal effects that can be found using algebraic methods by solving for a set of equations involving the causal effects and the covariances but cannot be identified using these methods.

The aim of this paper is thus to provide an extended set of graphical criteria for the identification of direct causal effects in linear SEMs. We will show examples where the desired causal effect can be identified using algebraic methods but cannot be identified by current graphical methods. Then based on the algebra involved in solving for these causal effects, we will propose this new set of graphical criteria, where the motivation is as follows. We will treat descendants of the cause variable as "path-specific instrumental variables", and find certain path-specific total effects which can be used to compute the desired causal effect. This extended set of graphical criteria we develop will thus check if these path-specific effects can be computed from covariance values that are available from observed data based on the graphical model.

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The outline of this paper is as follows. First, we provide the preliminary definitions for this paper, including linear SEMs, graphical models, statistical terms such as covariances, Wright's method of analysis, and the identification problem. Then, we recap previous results in solving the parameter identification problem, most notably IV-pairs [Brito and Pearl 2002a] and accessory sets [Tian 2007b]. Next, we present our new results for the identification of direct causal effects in linear SEMs based on an extended set of graphical criteria, which are able to identify direct causal effects which are not possible using previous results based on graphical criteria. Finally, we conclude our paper and provide a few discussions on future work on this topic.

2 Preliminaries

In statistical causal analysis, a *directed acyclic graph* (DAG) that represents cause-effect relationships is called a *path diagram*. A *directed graph* is a pair $G = (\boldsymbol{V}, \boldsymbol{E})$, where $\boldsymbol{V}$ is a finite set of vertices and the set $\boldsymbol{E}$ of directed edges is a subset of the set $\boldsymbol{V} \times \boldsymbol{V}$ of ordered pairs of distinct vertices. Regarding the graph theoretic terminology used in this paper, for example, refer to Pearl [Pearl 2000] and Spirtes et al. [Spirtes 2000].

Suppose a directed acyclic graph $G = (\boldsymbol{V}, \boldsymbol{E})$ with a set $\boldsymbol{V} = \{V_1, \cdots, V_n\}$ of variables is given. The graph G is called a path diagram, when each child-parent family in the graph G represents a linear structural equation model (SEM):

$$V_i = \sum_{V_j \in \mathrm{pa}(V_i)} \alpha_{v_i v_j} V_j + \epsilon_{v_i} \qquad i = 1, \ldots, n, \quad (1)$$

where $\mathrm{pa}(V_i)$ is a set of parents of V_i and $\alpha_{v_i v_j} (\neq 0)$ is called a *direct causal effect*. In addition random disturbances $\epsilon_{v_1}, \ldots, \epsilon_{v_n}$ are assumed to be normally distributed with mean 0. Here, when ϵ_{v_i} is correlated with ϵ_{v_j} $(i \neq j)$, this relationship is represented by a bi-directed (dashed) arc between X_i and X_j in the graph G.

Given a graph G, we define a *path* between the variables X and Y as a sequence of variables, $(V_0 = X, V_1, \ldots, V_n = Y)$, where there is an edge between each V_i and V_{i+1} and each variable appears only once in the sequence. A path is a *directed path* from X to Y if all edges between V_i and V_{i+1} are directed edges from V_i to V_{i+1}.

We say that V_i is a *collider* in the path if both the edges between V_{i-1} and V_i and between V_i and V_{i+1} point into V_i. If there are no colliders in the path, we say that the path is an *unblocked path* between X and Y. Given a set of variables $\boldsymbol{Z}$, we say the path is an *open path* if for all variables V_i which are not colliders on the path, $V_i \notin \boldsymbol{Z}$, and for all variables V_i which are colliders on the path, $V \in \boldsymbol{Z}$ where V is either V_i or a descendant of V_i. If there are no open paths between X and Y given $\boldsymbol{Z}$, we say that X and Y are *d-separated* given $\boldsymbol{Z}$. Otherwise, we say that they are *d-connected*.

The conditional independence induced from a set of equations in the form of Equation 1 can be obtained from the graph G according to d-separation [Pearl 2000], that is, when $\boldsymbol{Z}$ d-separates X from Y in a path diagram G, X is conditionally independent of Y given $\boldsymbol{Z}$ in the corresponding linear SEM [Spirtes 2000]. In this paper, it is assumed that a path diagram G and the corresponding linear SEM are faithful to each other; that is, the conditional independence relationships in the linear SEM are also reflected in G, and vice versa [Spirtes 2000].

Here, we denote some notations for further discussion. Let $\sigma_{xy \cdot z} = \mathrm{cov}(X, Y | \boldsymbol{Z} = \boldsymbol{z})$, $\sigma_{yy \cdot z} = \mathrm{var}(Y | \boldsymbol{Z} = \boldsymbol{z})$ and $\beta_{yx \cdot z} = \sigma_{xy \cdot z} / \sigma_{xx \cdot z}$ be a *conditional covariance* between X and Y given $\boldsymbol{Z} = \boldsymbol{z}$, a *conditional variance* of Y given $\boldsymbol{Z} = \boldsymbol{z}$ and the *regression coefficient* of x in the regression model of Y on x and $\boldsymbol{z}$, respectively. When $\boldsymbol{Z}$ is an empty set, they are omitted from these arguments.

A *total effect* τ_{yx} of X on Y is defined as the total sum of the products of the direct causal effects on the sequence of directed edges along all directed paths from X to Y. In addition, $\gamma_{yx} = \sigma_{xy} - \tau_{yx}$ is called a *spurious correlation* between X and Y.

A *path-specific total effect* is defined as the total sum of the product of the direct causal effects on the sequence of directed edges along directed paths of our interests from X to Y. For example, we define $\tau_{yx \cdot z}$ as the path-specific effects where all paths that pass through any variable in $\boldsymbol{Z}$ are not counted. Similar terms such as a *path-specific correlation* and a *path-specific spurious correlation* can be defined similarly.

Wright's method of path analysis [Wright 1934], which plays an important role in this paper, can be used to compute the covariance of two variables X and Y given a graph G. If the set S contains all paths $path = (V_0 = X, V_1, \ldots, V_n = Y)$ that are unblocked paths between

X and Y, we have:

$$\sigma_{xy} = \sum_{path} p_{v_0 v_0} \prod_{i=0,\ldots,n-1} p_{v_i v_{i+1}}, \quad (2)$$

where $p_{v_i v_{i+1}}$ is the parameter of the edge between V_i and V_{i+1}, which is either $\alpha_{v_{i+1} v_i}$ (or $\alpha_{v_i v_{i+1}}$) if it is a directed edge, or $\gamma_{v_{i+1} v_i}$ if it is a bi-directed edge. We define $p_{v_0 v_0}$ as $\sigma_{v_0 v_0}$ if all edges in *path* are directed edges, or 1 otherwise.

Given the matrix of observed covariances, we say that a causal parameter, such as a total effect and a direct causal effect, is *identified* if there is a unique solution of this parameter given the covariances. If all direct causal effects can be identified, we say that the model is identified.

The *single door criterion* is one of the famous graphical identification conditions for the direct causal effects, that is, the direct causal effect α_{yx} of X on Y is identifiable and is equal to $\beta_{yx \cdot z}$, if there exist a set $\boldsymbol{Z}$ of variables such that (i) $\boldsymbol{Z}$ contains no descendant of Y, and (ii) $\boldsymbol{Z}$ d-separates X from Y in $G_{X \to Y}$, formed by removing $X \to Y$ from G [Pearl 2000]. A set $\boldsymbol{Z}$ of variable satisfying both (i) and (ii) is said to satisfy the single door criterion relative to (X, Y).

3 Previous Results on Parameter Identification

There have been many work done on the problems of model identification and parameter identification using graphical test [Pearl 2000, Brito and Pearl 2002a, Brito and Pearl 2002b, Brito and Pearl 2002c, Tian 2004, Tian 2005]. Here we will focus only on the problem of parameter identification, in particular the identification of direct causal effects.

The previous most general result for the graphical identification of direct causal effects is the use of an *IV-pair* [Brito and Pearl 2002a], which embraces both *instrumental variables* [Bowden and Turkington 1984] and regression methods.

Lemma 1 *Let the graph G contain the directed edge $X \to Y$, and let W be a variable. Given $\boldsymbol{Z}$, a (possibly empty) set of variables which consists of non-descendants of Y and distinct from W, we say that W can be used as an instrumental variable given $\boldsymbol{Z}$ if the following two conditions are satisfied:*

1. *In the graph $G_{X \to Y}$, formed by removing $X \to Y$ from G, W and Y are d-separated given $\boldsymbol{Z}$;*

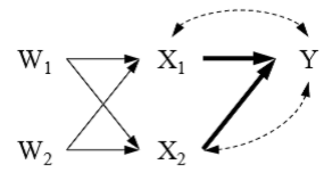

Figure 1: A path diagram where a multiple IV-pair are necessary to identify the direct causal effects of X_1 (or X_2) to Y.

2. *In the graph G, W and X are d-connected given $\boldsymbol{Z}$, or $W = X$.*

If the above conditions are satisfied, the direct causal effect α_{yx} is given by:

$$\alpha_{yx} = \frac{\sigma_{wy \cdot z}}{\sigma_{wx \cdot z}}. \quad (3)$$

The pair $(W, \boldsymbol{Z})$ can also be called an IV-pair.

Some identifiable direct causal effects cannot be found using a single IV-pair, but by the collective action of a multiple IV-pair, such as the example in Figure 1. We now define the conditions where a set of direct causal effects can be identified using multiple IV-pairs. The following lemma is adapted from previous work, where the multiple IV-pair are called *accessory sets* [Tian 2007a].

Lemma 2 *Let the graph G contain the directed edges $X_1 \to Y, \ldots, X_k \to Y$, and let $(\{W_1, \ldots, W_k\}$ be a list of variables. Given $\boldsymbol{Z}$, a (possibly empty) set of variables which consists of non-descendants of Y and distinct from $W_1, \ldots, W_k$, we say that $W_1, \ldots, W_k$ can be used as instrumental variables given $\boldsymbol{Z}$ if the following three conditions are satisfied:*

1. *In the graph $G_{X_1 \to Y, \ldots, X_k \to Y}$, formed by removing $\{X_1 \to Y, \ldots, X_k \to Y\}$ from G, $\{W_1, \ldots, W_k\}$ and Y are d-separated given $\boldsymbol{Z}$;*

2. *In the graph G, each pair of W_i and X_i are d-connected given $\boldsymbol{Z}$, or $W_i = X_i$, where we denote this path as $path_i$;*

3. *If two different paths, $path_i$ (from W_i to X_i) and $path_j$ (from W_j to X_j), have a common variable U, then either both $path_i[W_i, \ldots, U]$ and $path_j[U, \ldots, X_j]$ point into U, or $path_j[W_j, \ldots, U]$ and $path_i[U, \ldots, X_i]$ point into U, but not both, i.e., U cannot be a collider. Here, $path_i[W_i, \ldots, U]$ is a sub-path between W_i and U included in $path_i$, and the similar notation is used for other paths.*

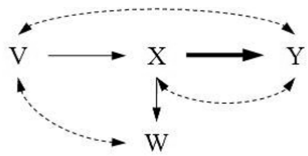

Figure 2: Path diagram for Example 1.

If the above conditions are satisfied, the direct causal effects $\alpha_{yx_1}, \ldots, \alpha_{yx_k}$ *can be solved by a system of equations involving equation coefficients:*

$$P_{i,j} = \sigma_{w_i x_j \cdot z},$$
$$Q_i = \sigma_{w_i y \cdot z},$$

where $Q_i = \sum_j P_{i,j}\alpha_{yx_j}$.

The last criterion is adapted from the *G criterion* [Brito and Pearl 2006] by adding the conditions necessary to deal with colliders. This criterion ensures that the system of equations we use to solve for the parameters $\alpha_{yx_1}, \ldots, \alpha_{yx_k}$ are linearly independent. It is a sufficient condition for parameter identifiability.

However, there are many cases where a single direct causal effect is identifiable even though neither IV-pair nor a set of variables satisfying the single door criterion can be found. We will illustrate this in the next section with a few examples, and extend previous results on graphical identification to find the direct causal effects.

4 New Results on Parameter Identification

Example 1 *Assume that cause-effect relationships between variables can be described as the DAG shown in Figure 2 and the corresponding linear SEM. The direct causal effects of* α_{xv} *and* α_{wx} *can be identified easily by the single door criterion [Pearl 2000]:*

$$\alpha_{xv} = \beta_{xv},$$
$$\alpha_{wx} = \beta_{wx \cdot v}.$$

We now want to find the direct causal effect of α_{yx}. *However, no IV-pair can be used to find* α_{yx} *using our previously shown results. Instead, we use Wright's method of path analysis (Equation 2). In particular, we have:*

$$\sigma_{wy} = (\gamma_{wv}\alpha_{yx} + \gamma_{yv}\alpha_{wx})\alpha_{xv} + (\sigma_{xx}\alpha_{yx} + \gamma_{yx})\alpha_{wx},$$
$$\sigma_{wx} = \gamma_{wv}\alpha_{xv} + \sigma_{xx}\alpha_{wx},$$
$$\sigma_{yx} = \gamma_{yv}\alpha_{xv} + \sigma_{xx}\alpha_{yx} + \gamma_{yx}.$$

Therefore, $\sigma_{wy} - \sigma_{yx}\beta_{wx\cdot v} = \gamma_{wv}\alpha_{yx}\alpha_{xv}$, *and* $\sigma_{wx} - \sigma_{xx}\beta_{wx\cdot v} = \gamma_{wv}\alpha_{xv}$. *Therefore,* α_{yx} *can be computed by:*

$$\alpha_{yx} = \frac{\sigma_{wy} - \sigma_{yx}\beta_{wx\cdot v}}{\sigma_{wx} - \sigma_{xx}\beta_{wx\cdot v}}.$$

Notice that in this example, neither W nor V can be used in any IV-pair to identify α_{yx}. In particular for W, the presence of an open path $W \leftarrow X \leftrightarrow Y$ in the graph $G_{X\to Y}$ makes it invalid to be used as an instrumental variable. However, if we consider a latent variable U along the bi-directed edge between W and X, this latent variable, if observable, can be used as an instrumental variable to identify α_{wx} [Cai and Kuroki 2008], meaning we need to aim to find a method to "indirectly" estimate the correlations both between U and X, and U and Y. To do this, we use W, which is a descendant of the cause variable X, as a "path-specific instrumental variable", and estimate the correlations both between W and X, and between W and Y, through certain paths in the graph. To proceed with our discussion, we make the following definition.

Definition 1 *Let the graph* G *contain the directed edge* $X \to Y$, *and let* W *be a descendant of* X *(but not* Y*). Given* $\boldsymbol{Z}$, *a (possibly empty) set of variables which consists of non-descendants of* X *and* Y *distinct from* W, *we say that* W *can be used as a "path-specific instrumental variable" given* $\boldsymbol{Z}$ *if the following two conditions are satisfied:*

1. *In the graph* G, W *and* X *are d-connected given* $\boldsymbol{Z}$, *where at least one open path between* W *and* X *given* $\boldsymbol{Z}$ *is not a directed path from* X *to* W.
2. *In the graph* $G_{X\to Y}$, *formed by removing* $X \to Y$ *from* G, W *and* Y *are either d-separated given* $\boldsymbol{Z}$, *or d-connected such that all open paths between* W *and* Y *given* $\boldsymbol{Z}$ *must contain a sub-path which is a directed path from* X *to* W.

The path diagram in Figure 2 satisfies the conditions in Definition 1. In particular, in the graph $G_{X\to Y}$, formed by removing $X \to Y$ from G, W and Y are d-connected only through the path $W \leftarrow X \leftrightarrow Y$, where it contains a directed path from X to W.

Now we are ready to compute the direct causal effect α_{yx}. We classify all open paths between W and Y given $\boldsymbol{Z}$ in the graph G into two subsets:

1. $S'_{wy\cdot z}$: Paths that contain a sub-path which is a directed path from X to W (a path-specific correlation between Y on W, denoted as $s'_{wy\cdot z}$);

2. $S_{wy\cdot z}$: Paths that do not contain a sub-path which is a directed path from X to W (a path-specific spurious correlation between Y and W, denoted as $s_{wy\cdot z}$).

Similarly, we classify all open paths between W and X given $\boldsymbol{Z}$ in the graph G into two subsets:

1. $S'_{wx\cdot z}$: Paths that are directed paths from X to W (a path-specific total effect of X on W, denoted as $s'_{wx\cdot z}$);

2. $S_{wx\cdot z}$: Paths that are not directed paths from X to W (a path-specific correlation between X and W, denoted as $s_{wx\cdot z}$).

First, the subsets $S'_{wy\cdot z}$ and $S_{wy\cdot z}$ are mutually exclusive, and so are the subsets $S'_{wx\cdot z}$ and $S_{wx\cdot z}$. Therefore, we have:

$$\begin{aligned} s'_{wy\cdot z} + s_{wy\cdot z} &= \sigma_{wy\cdot z}; \\ \sigma_{xx\cdot z} s'_{wx\cdot z} + s_{wx\cdot z} &= \sigma_{wx\cdot z}. \end{aligned}$$

Second, $S'_{wx\cdot z}$ contains all open directed paths from X to W given $\boldsymbol{Z}$ in the graph G. Since $\boldsymbol{Z}$ does not contain any descendants of X, this is equivalent to all open directed paths from X to W. Therefore, we have:

$$s'_{wx\cdot z} = \tau_{wx}.$$

Third, any open path between X and Y given $\boldsymbol{Z}$ in the graph G must not pass through W. Otherwise, as the sub-path between W and Y does not pass through X, this would violate the second condition of Definition 1. Moreover, as all paths in $S'_{wx\cdot z}$ are directed paths from X to W, they do not point into X. This means a path in $S'_{wy\cdot z}$ can be (and must be) formed by extending a path in $S'_{wx\cdot z}$ by an open path between X and Y given $\boldsymbol{Z}$. Therefore, we have:

$$s'_{wy\cdot z} = \sigma_{yx\cdot z} s'_{wx\cdot z} = \sigma_{yx\cdot z}\tau_{wx}.$$

Fourth, as all paths in $S_{wy\cdot z}$ do not contain a sub-path which is a directed path from X to W, they must contain the directed edge $X \to Y$, otherwise this would violate the second condition of Definition 1. This means a path in $S_{wy\cdot z}$ can be (and must be) formed by extending a path in $S_{wx\cdot z}$ by the directed edge $X \to Y$. Therefore, we have:

$$s_{wy\cdot z} = \alpha_{yx} s_{wx\cdot z}.$$

Finally, the first condition of Definition 1 (and the faithfulness condition) ensures that $s_{wx\cdot z} \neq 0$. Combining all equations, we have the following theorem.

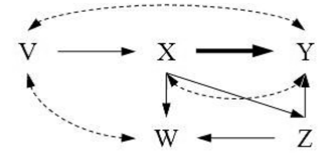

Figure 3: Path diagram which satisfies the conditions of Definition 1.

Theorem 1 *Given a path diagram which satisfies the conditions in Definition 1, the direct causal effect* α_{yx} *is given by:*

$$\alpha_{yx} = \frac{\sigma_{wy\cdot z} - \sigma_{yx\cdot z}\tau_{wx}}{\sigma_{wx\cdot z} - \sigma_{xx\cdot z}\tau_{wx}}. \tag{4}$$

Theorem 1 (Equation 4) expands on the the use of the IV-pair in Lemma 1 (Equation 3), by allowing the use of the "path-specific instrumental variable" W which is a descendant of X, showing that the direct causal effect α_{yx} is identifiable as long as the total effect of X on W given $\boldsymbol{Z}$, τ_{wx}, is identifiable.

For example, in the path diagram in Figure 2, the total effect of X on W is given by $\tau_{wx} = \alpha_{wx} = \beta_{wx\cdot v}$, and we can use Equation 4 to compute α_{yx}. For another example, in the path diagram in Figure 3, given Z, W can be used as an "path-specific instrumental variable" for the direct causal effect of $X \to Y$, and the total effect of X on W is given by $\tau_{wx} = \beta_{wx\cdot vz}$.

We conclude this section by stating that we can use a set of multiple "path-specific instrumental variables" to identify multiple direct causal effects similar to Lemma 2, as long as the G criterion is satisfied. However, we do not state the details here.

5 Discussion and Conclusion

In this paper, we presented our new results for the identification of direct causal effects in linear SEMs based on an extended set of graphical criteria, which are able to identify direct causal effects which are not possible using previous results based on graphical criteria. The results are useful in the sense that they allow us to effectively identify the desired direct causal effect, by first checking whether the model satisfies the graphical criteria, then using the necessary covariance values and other total effects to compute the causal effect, instead of solving the whole set of equations for all variables given by Wright's method of analysis.

In future work, there are three main areas which we will pursue:

Algorithm We need to find an algorithm where we can easily test for the satisfaction of our graphical criteria given in this paper. Previous work on IV-pairs and accessory sets use flow analysis to find the necessary variables, and we look to adapt these algorithms for our extended set of graphical criteria.

Completeness It remains to be seen if the graphical criteria given in this paper, combined with previous methods such as IV-pairs, are complete, i.e., they are necessary conditions for the identification of causal effects in linear SEMs. If not, we will look for counterexamples, and if possible, extend further our graphical criteria. In particular, our new graphical criteria are dependent on a latent variable U, if observable, being a valid instrumental variable for our direct causal effect. Recent work has looked at the role of a latent variable in identifying causal effects [Cai and Kuroki 2008].

Robustness Even in models where previous graphical criteria, such as instrumental variables, can be used to identify a certain causal effect, our new set of criteria may provide another distinct function for computing this causal effect. The concept of *robustness* [Pearl 2004] deals with whether a function for computing a causal effect is still valid when certain independence relations are relaxed in a model (by adding edges between variables). If for all super-graphs of our current graph, function A is valid whenever function B is valid, then we say that function A is at least as robust as function B, and should at leat be more preferred than function B, because function A will remain valid even in cases where function B is no longer valid when some of our current independence relations are relaxed. Moreover, if we are given two functions that are no more robust than one another, then the computation of the desired causal effect using these two different functions (and the fact that the two computations agree) will greatly confirm the correctness of our model. Therefore, we would like to compare the robustness quality of our new set of criteria compared with previous graphical criteria.

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